

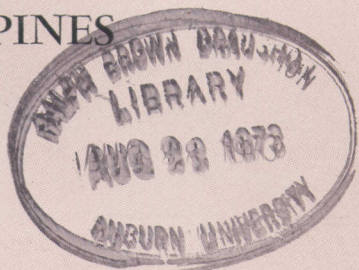
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RELATIONSHIP BETWEEN POINT  
DENSITY MEASUREMENTS AND  
SUBSEQUENT GROWTH OF  
SOUTHERN PINES



*Alabama*  
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AGRICULTURAL EXPERIMENT STATION  
AUBURN UNIVERSITY

R. Dennis Rouse, Director

Auburn, Alabama

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# Relationship Between Point Density Measurements and Subsequent Growth of Southern Pines

E. W. JOHNSON<sup>1</sup>

## INTRODUCTION

TREES growing in forest stands necessarily compete with one another for sunlight, air, water, and soil nutrients. The degree of competition varies from tree to tree and is dependent on a host of interacting variables. No practical way has been found to assess all these factors to arrive at a measure of the competitive pressure exerted against an individual tree. However, attempts have been made to approach this problem indirectly through the use of a number of different concepts or ideas. These procedures yield measures of what has been named "point density."<sup>2</sup> This bulletin describes some of these procedures and the result of tests in which the procedure results were correlated with subsequent periodic annual increment in diameter breast high (d.b.h.) and basal area for loblolly (*Pinus taeda*, L.), longleaf (*P. palustris*, Mill.), and slash (*P. elliotti*, Engelm.) pines grown in plantations in east-central Alabama.

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<sup>1</sup> Professor, Department of Forestry.

<sup>2</sup> Spurr (1962) defined point density as "the stand basal area as measured at a given point within the stand rather than over a given area." This definition is somewhat limited since the term appears to be quite suitable for a number of methods, not involving basal area, that can be used to estimate the competitive pressure against individual trees. Consequently, in this paper the term "point density" will be used for all the measures considered.

## METHODS OF EXPRESSING POINT DENSITY

### Competing Basal Area Per Unit Ground Area

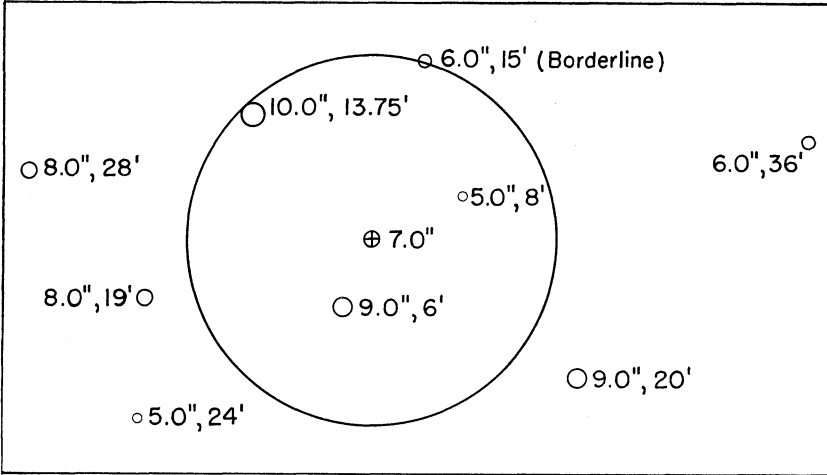
This is the most commonly used measure of stand density. It is determined by summing the basal areas of the trees growing on a plot of land of a given size. Usually it is expressed in square feet of basal area per acre, or square meters per hectare.

Conventionally, basal area per unit ground area is obtained from a relatively large plot (0.2 to 0.25 acre) and is not a good measure of point density or competitive pressure on any particular tree in the plot because the procedure merely averages conditions over the entire plot. Since few stands are uniformly stocked, the pressure of competition will differ from tree to tree within the plot, depending on the degree of clumping of the trees and the position of the sample tree with respect to nearby trees.

A procedure that would tend to overcome this averaging problem is to use a small, fixed-radius plot *centered on the sample tree*. Figure 1 and Appendix A.1. show how such a fixed-radius plot might be used. Steneker and Jarvis (28) employed this procedure to provide them with one of the several competition indices they used while studying the effect of competitive pressure on individual white spruces (*Picea glauca* (Moench.) Voss). Plots with radii of 25 feet were used in their study. The optimum plot size would necessarily depend on a number of variables and probably could be determined only through empirical means. However, it seems logical to assume that the trees growing closest to the sample tree would exert the greatest competitive pressure. Consequently, the plot size should be large enough to include as many as possible of these but not so large as to include many trees of negligible influence. Thus, in dense stands of small trees, the plots probably should be kept small while in more open stands, or in stands made up of larger trees, the plots probably should be relatively large.

In a situation such as this it is possible to compute the basal area per unit land area in either one or two ways. One can either include or exclude the basal area of a sample tree. It would appear logical to exclude the sample tree, since one is interested in the pressure against that tree and not in the total basal area on the plot.

The idea of varying plot size in proportion to tree size leads directly to the "angle-count" method of estimating basal area per



Scale of diameters twice that of location.

Plot radius = 15.0 feet

Basal area/acre, excluding sample tree = 75.2 sq. ft.

, including sample tree = 91.7 sq. ft.

Steneker-Jarvis competition indices:

- $I_{SJ1} = 0.017$
- $I_{SJ2} = 0.369$
- $I_{SJ3} = 21.122$
- $I_{SJ4} = 2.586$

FIG. 1. Example of the use of fixed radius plots, centered on the sample tree, to obtain basal area per acre and the Steneker-Jarvis competition indices  $I_{SJ1}$ ,  $I_{SJ2}$ ,  $I_{SJ3}$ , and  $I_{SJ4}$ . Computations shown in Appendices A. 1, and A. 2.

unit of land area, originated by Bitterlich (3) and further developed by Grosenbaugh (8,9). In this approach to point density measurement, the sample tree may be considered to be at the center of an infinite number of concentric circular plots. Each competing tree is associated with one of these plots through the relationship:

$$BAF = \frac{C_r^2}{R^2} \dots\dots\dots (1)$$

where BAF = "Basal area factor" = the number of square units of basal area (square feet, square meters, etc.) per unit of land area (acre, hectare, etc.) represented by one tree;

$C = 43,560$  when stand density is in terms of square feet of basal area per acre. It is 10,000 when stand density is expressed in terms of square meters of basal area per hectare;

$r = \frac{\text{d.b.h.}}{2}$  = radius of competing tree, in the same units as  $R$ ;

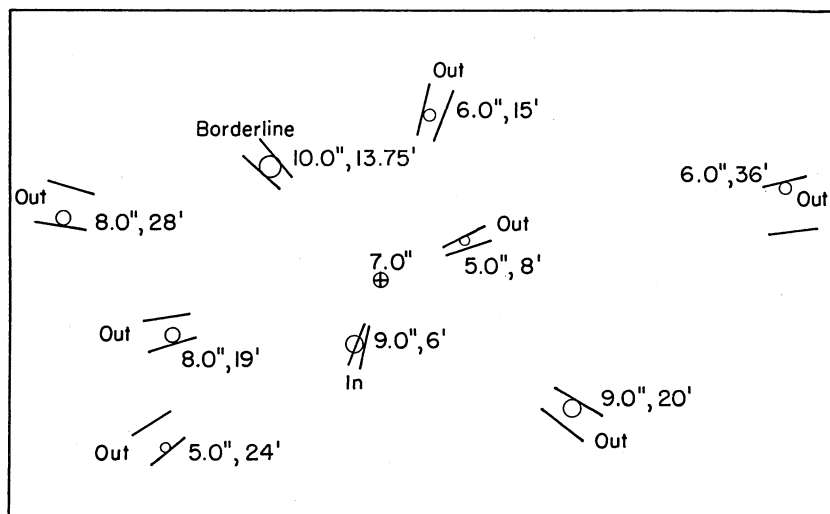
$R$  = radius of plot associated with the competing tree.

The basal area factor (BAF) is arbitrarily chosen (e.g., 10 square feet of basal area per acre). Any tree whose distance ( $R$ ) from the center of the sample tree is less than:

$$R = \sqrt{\frac{C r^2}{\text{BAF}}} \dots\dots\dots (2)$$

is assumed to be competing with the sample tree and the measure of its competition is equal to the BAF. Consequently, the sum of these BAF values for the competing trees is a measure of the competitive pressure against the sample trees expressed in basal area per unit of land area. Figure 2 shows the angle-count method being used with the same sample tree and surrounding stand depicted in Figure 1. As with the fixed-radius plots, it is possible to estimate the basal area per unit land area including or excluding the sample tree and again it would appear logical to exclude the sample tree.

A variety of basal area factors can be used in measurements of this type. However, one would expect that the smaller the BAF the more the measure of point density would represent average stand conditions rather than the conditions immediately adjacent to the sample tree. Conversely, the larger the BAF, the more specific would be the measure of point density. This reasoning, however, is not supported by the results of a study carried out by Lemmon and Schumacher (17,18) in ponderosa pine (*Pinus ponderosa* Laws). They used four different basal area factors (10, 20, 30, and 40 square feet per acre), and the response variable was periodic annual volume increment in cubic feet. The 10 square feet per acre BAF yielded the highest correlation between point density and increment. The reasons for this divergence from theoretical results are not known.



Scale of diameters twice that of location.

Basal area factor = 40 sq. ft. basal area/acre/tree

Sweeping angle =  $3^{\circ} 28'$  (Angle shown is to above scales and thus is  $6^{\circ} 56'$ ).

Count of "in" trees =  $1\frac{1}{2}$ , excluding sample tree  
 =  $2\frac{1}{2}$ , including sample tree

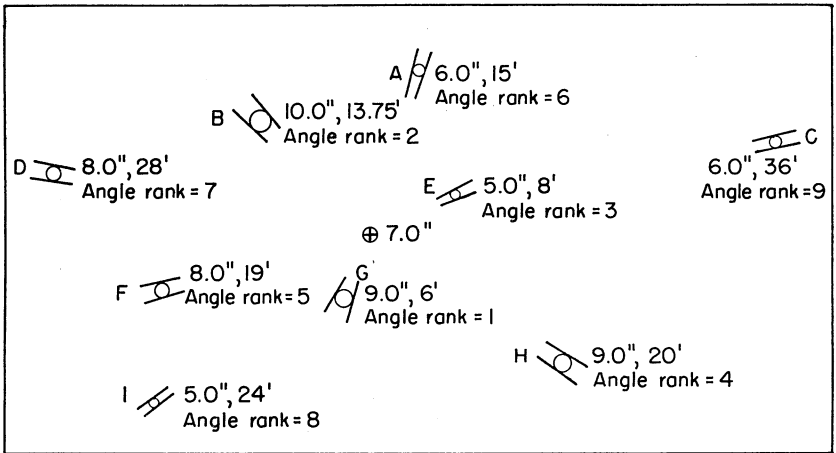
Basal area/acre = 60 sq. ft., excluding sample tree  
 = 100 sq. ft., including sample tree

FIG. 2. Example of the use of Bitterlich's angle-count method of obtaining basal area per acre centered on the sample tree.

Spurr (26) originated a variant of the angle-count method, which he called the "angle-summation" method. In this procedure (see Figure 3 and Appendix A.3.) the angles subtended by the trees surrounding the sample tree are measured or computed, then ranked in magnitude. An arbitrarily chosen number of the highest ranked trees is used in the subsequent computations (e.g., if four trees are to be used the four trees subtending the largest angles are used). An estimate of basal area per unit of land area is made, first assuming that the tree subtending the largest angle is an exact borderline tree with only half of its basal area within the plot. The basal area per unit of land area is computed by using a modification of the basic formula used in the Bitterlich method:

$$B_1 = \frac{0.5 C r_1^2}{R_1^2} \text{-----} (3)$$

where:  $B_1$  = estimate of basal area per unit of land area



Scale of diameters twice that of location

Using four trees

Basal area/acre = 68.12 sq. ft., excluding sample tree  
 = 131.87 sq. ft., including sample tree

FIG. 3. Example of the use of Spurr's angle-summation method of obtaining basal area per acre centered on the sample tree. (Computations shown in Appendix A. 3.)

based on the tree subtending the largest angle;

0.5 = expansion factor (Since only half the tree is inside the plot, only half its basal area contributes to the basal area per unit of land area.);

$r_1$  = radius of highest competing tree;

$R_1$  = distance between sample tree and highest ranked competitor.

Then a second estimate of the basal area per unit of land area is made assuming that the tree subtending the second largest angle is an exact borderline tree. The basal area per unit of land area is computed as follows:

$$B_2 = \frac{1.5 C r_2^2}{R_2^2} \dots\dots\dots (4)$$

where:  $B_2$  = estimate of basal area per unit of land area based on the two trees subtending the largest angles;

1.5 = expansion factor (All of the first tree and half



of the second tree are contributing to the basal area per unit of land area.);

$r_2$  = radius of second highest ranked competitor;

$R_2$  = distance between sample tree and second highest ranked competitor.

This procedure is repeated with succeeding trees until the desired number is reached. All of these estimates are then averaged, yielding the point density value in terms of basal area per unit of land area. As in the case of the preceding methods, the sample tree may be excluded or included and probably should be excluded.

Figure 4 shows the pattern of change in the magnitude of the estimates of basal area per unit of land area, for the same stand shown in Figure 3, with different numbers of competing trees involved and the sample tree excluded. This pattern of change is associated with situations where the differences between the values of  $r^2/R^2$  for successively ranked trees are relatively large. This occurs in nonuniform stands with wide ranges in stem diameters and highly variable distances between trees, as is the case in the stand being used as an example. In more uniform stands the pattern of change is reversed so that the estimate of basal area per unit of land area increases as the number of competing trees included in the computations increases. Spurr tested the angle-summation procedure using data from a Douglas fir (*Pseudotsuga menziesii* (Mirb. Franco)) plantation in New Zealand and found the rising pattern. He attributed the rise to the exclusion of the sample tree from the estimates. This exclusion would make the estimates of basal area per unit of land area too small. When only one competitor is used, this negative bias will be relatively large. As the number of competitors used in the computations increases, the effect of the exclusion of the sample tree becomes less and less and the estimates become larger and larger. This is sound reasoning and the phenomenon undoubtedly occurs in all cases where the sample tree is excluded. However, if the stem diameter and tree spacing are sufficiently irregular, the typical rising of the basal area estimates may be overridden to produce a downward trend.

The pattern of rising or falling of the basal area estimates is of importance to the angle-summation method only in that it provides a basis for choosing the number of competitors which

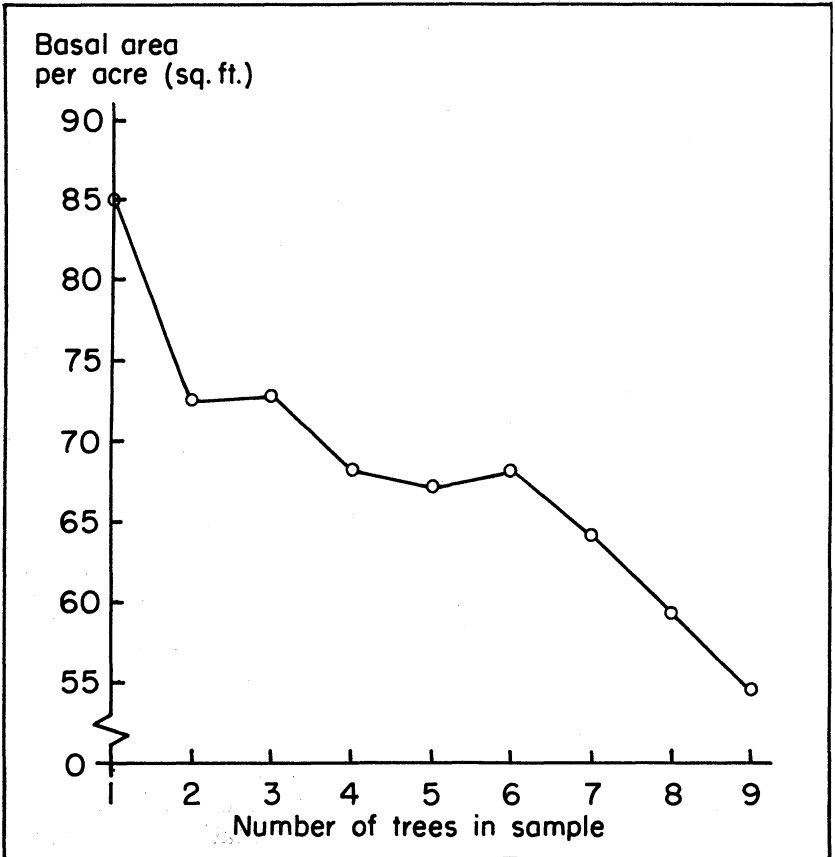
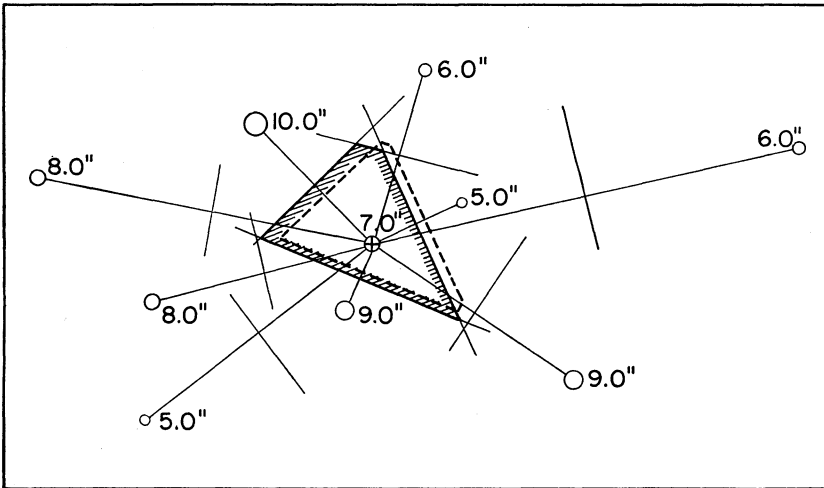


FIG. 4. Relationship between the estimate of the basal area per acre and the number of trees used in Spurr's angle-summation method.

should be used in the estimate. Theoretically, the basal area estimate should stabilize at or near the size sample that yields the actual basal area per unit of land area of the entire stand. This point of stabilization can be used as a guide to the number of trees needed for an estimate. If one desires to measure *point* rather than *stand* density he should use a sample size that is smaller than that at which stabilization occurs. Spurr, using data from the Douglas fir plantation in New Zealand, found that stabilization began to occur when approximately 9 trees were used.

#### Growing Space (Area) Available to Tree

Brown (4) has devised a method of expressing point density in terms of the ground area that could be assigned to the sample



Scale of diameters twice that of location  
 The solid line shows area defined by Brown's Method.  
 The dashed line shows area defined by the modification.  
 Area potentially available to sample tree: 105.75  
 Brown's method = 105.75 sq. ft.  
 Modification of Brown's method = 96.01 sq. ft.

FIG. 5. Growing space or "area potentially available" to sample tree using Brown's method and a modification of his method.

tree. This is done by first connecting the sample tree to all the surrounding trees with line segments, Figure 5. The smallest closed figure, or polygon, formed by the perpendicular bisectors of these line segments encloses the ground area that is assigned to the sample tree. Area of the polygon is an inverse measure of competitive pressure.

### Competitive Influence Zone Overlap

The space that a tree occupies is three-dimensional. This space may be thought of as an irregularly shaped "solid" that extends vertically from the deepest root to the tip of the bole and horizontally, aboveground, to the tips of the branches and, underground, to the tips of the widest spread roots. Only in the case of isolated, free-growing trees does this space reach its maximum potential size. This is termed maximum potential growing space (M.P.G.S.), whose magnitude is directly proportional to size of the tree. Furthermore, evidence indicates that the horizontal extent of the M.P.G.S. probably is greater underground than it is above ground (10,11,21,23,25).

If the M.P.G.S. of any other plant (tree or otherwise) encroaches on that of a given tree, competition for the overlapping space probably occurs. Several methods of expressing point density that are based on the idea of measuring the amount of overlap of these growing spaces have been devised.

A direct evaluation of the volume of overlap between M.P.G. spaces is not possible because there is no way of knowing what the bounds of the spaces would have been if no competition existed. The best that can be done is to use a mathematical model that approximates the actual situation. One such model can be developed by assuming that the M.P.G.S. is a right circular cylinder, centered on the tree, with an end area equal to the horizontal cross-sectional area of the actual M.P.G.S. and that the cylinder has a total altitude equal to the total vertical length of the actual M.P.G.S. Any overlap or interpenetration constitutes an estimate of that competition. The vertical dimension of the interpenetration in a model of this type is of little significance since a right cylinder is a poor approximation of the actual vertical configuration of the M.P.G.S. Consequently, with this model, it is logical to ignore the vertical dimension and to use the magnitude of the overlap between horizontal cross-sections of the right circular cylinders as a measure of competitive pressure. These horizontal cross-sections were named "competition circles" by Staebler (27), "zones of influence" by Opie (22), and "competitive influence zones" by Bella (1,2). Bella's term will be used in this report.

Before overlaps of competitive influence zones (C.I.Z.'s) can be measured, it is necessary to define the sizes of the circles. Their areas should be equal to the areas of the maximum horizontal cross-sections of the M.P.G. spaces, which probably would involve root extent rather than crown spread. Since root extent cannot be determined in a non-destructive manner, studies involving the evaluation of tree growth following point density assessment must be based on the use of approximations rather than actual C.I.Z. areas. In the absence of firm information about root extent, the best indicator of the size of the C.I.Z. is crown spread. However, crown spread itself is strongly influenced by competition, which means that its correlation with actual C.I.Z. may be quite poor. Workers in the field of point density evaluation have approached the problem of C.I.Z. extent in several ways. These will be mentioned as each worker's procedures are described.

Staebler (27), working with Douglas fir, apparently was the first

to use the concept of overlapping C.I. Zones to evaluate competitive pressure against individual trees. To express zone size, he related zone diameter to tree diameter through the simple linear function:

$$A = a (D) + k \dots \dots \dots (5)$$

where: A = diameter of C.I. Zone, in feet;

D = d.b.h.o.b. of tree, in inches;

a = arbitrarily chosen multiplying coefficient (values used were 0.8, 1.2, and 1.9);

k = arbitrarily chosen y intercept (values used were 3, 5, and 7).

Staebler's basis for this model was the "D times" and "D plus" relationships sometimes used in thinning. Staebler considered the *area of overlap* of C.I.Z. circles to be the most desirable measure of competition. However, his opinion was that the mathematical expression required to compute this area was too complicated (he did this work prior to the widespread availability of electronic computers). Consequently, he discarded the idea of area overlap and, instead, used the length of the portion of the line connecting the centers of the two circles and lying within both circles, Figure 6 and Appendix A.4. If more than one competitor was involved, the sum of the lengths would be the measure of competition or point density. Staebler referred to this sum as an "index of competition." Its formula is:

$$I_{S1} = \sum_{i=1}^n d_i \dots \dots \dots (6)$$

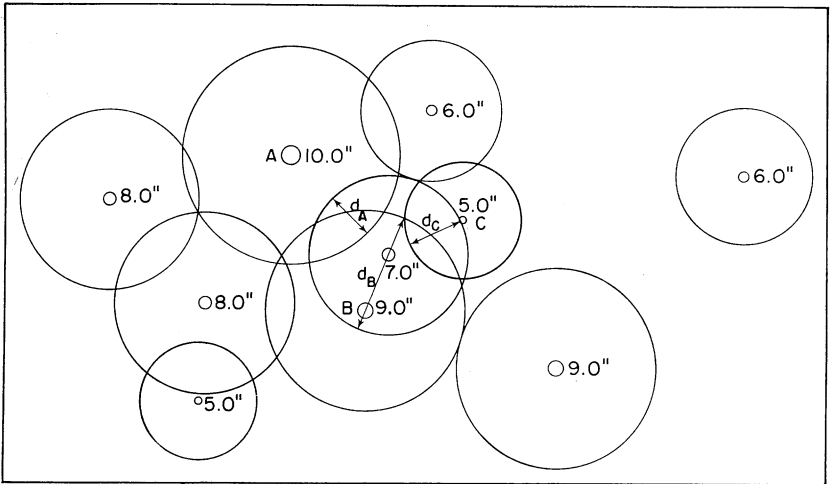
where:  $I_{S1}$  = index of competition;

$d_i$  = length of line segment within the circles of the sample tree and the *ith* competitor;

n = number of competing trees.

Staebler recognized further than a single large overlap would indicate a greater degree of competition than would an equal sum of several short overlaps. To compensate for this difference in competition he developed a second index of competition ( $I_{S2}$ ), the sum of the squared overlaps:

$$I_{S2} = \sum_{i=1}^n d_i^2 \dots \dots \dots (7)$$



Scale of diameters twice that of locations and C.I. Zones.

Radius of C.I.Z. in feet = d.b.h. of tree in inches + 1.

$$I_{S1} = 23.2 \text{ feet}$$

$$I_{S1}/F = 0.86$$

$$I_{S2}/F = 7.67$$

$$I_{S3}/F = 7.04$$

$$I_{S4}/F = 65.05$$

$$I_{S5} = 145. \%$$

**FIG. 6.** Competitive influence zone overlap using Staebler's competition indices  $I_{S1}$ ,  $I_{S1}/F$ ,  $I_{S2}/F$ ,  $I_{S3}/F$ ,  $I_{S4}/F$ , and  $I_{S5}$ . ( $I_{S5}$  is explained in the section "Point Density Expressions Tested." See Appendix A. 4. for computations.)

To compensate for tree size differentials he developed a third index ( $I_{S3}$ ) which was the sum of the products of the overlaps and the d.b.h.'s of the competing trees:

$$I_{S3} = \sum_{i=1}^n (d_i D_i) \dots\dots\dots (8)$$

where:  $D_i$  = d.b.h. of the  $i$ th competing tree.

In good measure, he also developed the index ( $I_{S4}$ ):

$$I_{S4} = \sum_{i=1}^n (d_i^2 D_i) \dots\dots\dots (9)$$

Staebler further recognized that a large sample tree in a given situation usually would have a larger index of competition than would a small sample tree. However, in the case of the larger

tree, the competition would be less severe because the tree had a higher degree of dominance over its neighbors. In any investigation relating growth to competitive pressure, this fact would have to be recognized. Staebler solved this problem by dividing each of his indices by an area proportional factor which he labelled "F."

$$F = \left[ \frac{a}{2} (D_s + D_a) + k \right]^2 / 10 \dots\dots\dots (10)$$

where: a = arbitrarily chosen multiplying coefficient (values used were 0.8, 1.2, and 1.9);

$D_s$  = d.b.h.o.b. of sample tree;

$D_a$  = d.b.h.o.b. of average tree in stand;

k = arbitrarily chosen y-intercept (values used were 3, 5, and 7).

He rounded F to the nearest digit.

Staebler tested his procedure by means of multiple linear regression. The dependent variable was the residual from the curve of d.b.h. growth over d.b.h. This dependent variable was chosen because it helped to compensate for the fact that large trees grow faster than small trees. The regression model he tested was:

$$y = a + b_1(I_{S1}/F) + b_2(I_{S2}/F) + b_3(I_{S3}/F) + b_4(I_{S4}/F) \dots\dots\dots (11)$$

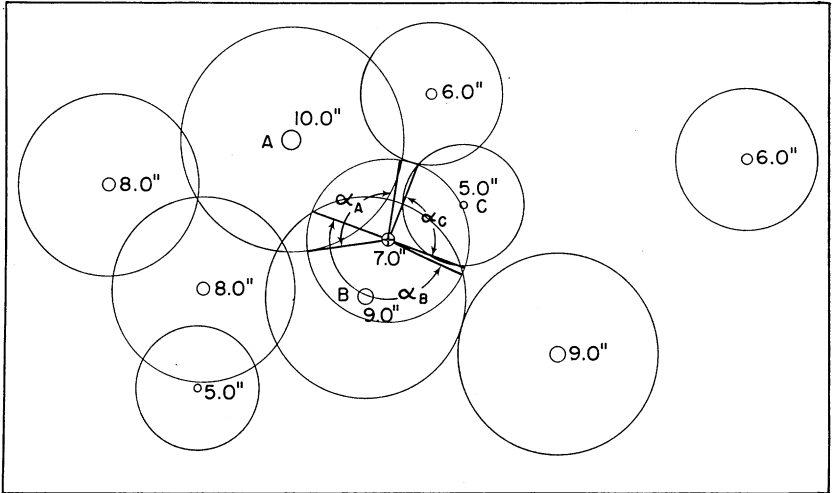
In spite of the foregoing elaborate and well-reasoned procedures, designed to compensate for dominance and competition differences, the best multiple correlation coefficient that Staebler obtained was only 0.575.

Newnham (20), in the course of developing a stand growth model for Douglas fir, devised a competition index that makes use of overlapping C.I. Zones. However, instead of using either the length of linear overlap or the area of overlap, he determined the proportion of the total circumference of the C.I.Z. of the sample tree that was occupied or overlapped by the C.I. Zones of the competing trees, Figure 7 and Appendix A.5.:

$$I_N = \frac{1}{2\pi} \sum_{i=1}^n [\alpha_i (A_i/A_s)] \dots\dots\dots (12)$$

$I_N$  = index of competition;

$\alpha_i$  = the angle, measured at the center of the C.I.Z. of the



Scale of diameters twice that of locations and C.I. Zones

Radius of C.I.Z., in feet = d.b.h., in inches,  $\div 1$

$I_N = 120\%$

$I_{N2} = 103\%$

**FIG. 7.** Competitive influence-zone overlap using Newnham's competition indices  $I_N$  and  $I_{N2}$ . ( $I_{N2}$  is explained in the Section "Point Density Expressions Tested." See Appendix A. 5. for computations.)

sample tree, subtended by the portion of the circumference overlapped by the  $i$ th competitor, in radians. If  $\alpha_i$  is expressed in degrees, 360 should be substituted for the  $2\pi$  term;

$A_i$  = diameter of the C.I.Z. of the  $i$ th competitor;

$A_s$  = diameter of the C.I.Z. of the sample tree.

The  $(A_i/A_s)$  term is a weighting factor used to take into account the relative sizes of the trees. A tree with a crown larger than another tree usually is also taller and has an additional competitive advantage.

Since it is possible for many C.I. Zones to overlap that of the sample tree it is possible for  $I_N$  to exceed 1.00, or 100 per cent.

Krajicek *et al* (13,14), in their development of the crown competition factor, made use of an idea, apparently first suggested by Lane-Poole (15), that the C.I.Z. is closely approximated by the crown-spread of open-grown trees and that this crown-spread is



closely related to d.b.h. This relationship can be established by using regression, e.g.:

$$A = a + b D, \text{ or some higher polynomial} \dots\dots\dots (13)$$

where: A = diameter of crown (i.e., C.I.Z.), in feet;

D = d.b.h.o.b., in inches;

a and b = regression coefficients.

By this procedure, the C.I.Z. for any tree can be approximated, regardless of competition, provided its d.b.h. is known.

Newnham (20) used this approach, with modification, to define C.I. Zones. He recognized that the actual C.I.Z. of a tree in a closed stand probably did not coincide with the crown spread of an open-grown tree of the same d.b.h. Furthermore, the lack of coincidence probably was a function of stand age and initial spacing. To overcome this problem he included the correction factor (K) in Equation 14:

$$A = a + b D K \dots\dots\dots (14)$$

Using empirical methods not clearly delineated, he developed a series of curves showing the value of K for different combinations of stand age and initial spacing. These values ranged from 0.6 to about 1.0, increasing with age and initial spacing. Within the context of his stand model, Newnham used this correction factor to compensate for changes in competitive pressure brought by mortality among the competitors.

Newnham's procedure can lead to some anomalies unless the investigator is careful to evaluate exactly what has occurred in each case. For example, Figure 8 shows a series of situations where a tree competes with a larger sample tree. Assume that this competing tree can be moved toward or away from the sample tree. When the two trees are separated so that their C.I.Z. circles are tangent (situation A) the angle  $\alpha$  is equal to zero and it would be assumed that no competition exists. If the competing tree is moved toward the sample tree (situations B and C) the angle  $\alpha$  increases in magnitude, correctly indicating increasing competitive pressure, and reaching a maximum when the overlap is as in situation D. However, if the convergence is continued,  $\alpha$  will begin to decrease (situation E). If continued still further, the C.I.Z. of the competitor will be brought entirely inside that of the sample tree (situation F). Since  $\alpha$  is intended to be a measure of competitive pressure, the pressure in situations E and

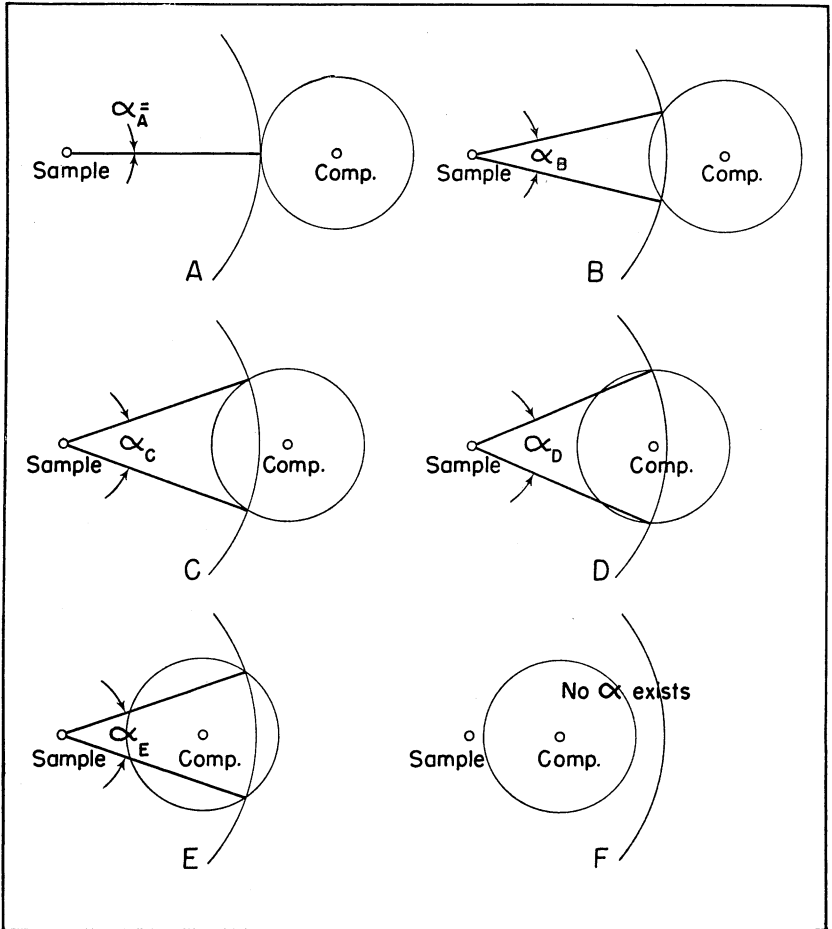


FIG. 8. Effect of size of competitive influence-zone and distance between competing trees on Newnham's competition indices when the competitor is smaller than the sample tree.

F could be mistakenly interpreted to be less than that at D. Newnham did not acknowledge this problem but apparently accepted the value of  $\alpha$  as computed, regardless of the situation. When situation F occurred, he assigned a value of zero to  $\alpha$ . Although Newnham's treatment of situations like E and F appears illogical, the effects on the end results probably were minimal. Any competitor small enough to occur under these situations probably exerts too little pressure to be of consequence.

When the sample tree is smaller than the competitor, the situa-

tions shown in Figure 9 may occur. Situations A, B, and C are similar to those already encountered. In situation D,  $\alpha$  reaches  $180^\circ$ , or  $\pi$  radians. As the distance between the competitors continues to decrease,  $\alpha$  increases to a maximum of  $360^\circ$ , or  $2\pi$  radians, then vanishes. Newnham accepted  $\alpha$  as shown except in the case of situation F, where he arbitrarily assigned a value of  $360^\circ$ , or  $2\pi$  radians. Except for the arbitrary assignment of a value in F, these actions are consistent with the theory. In the case of situation F, some recognition should be made of differences in separation distances between competitors. However, this could

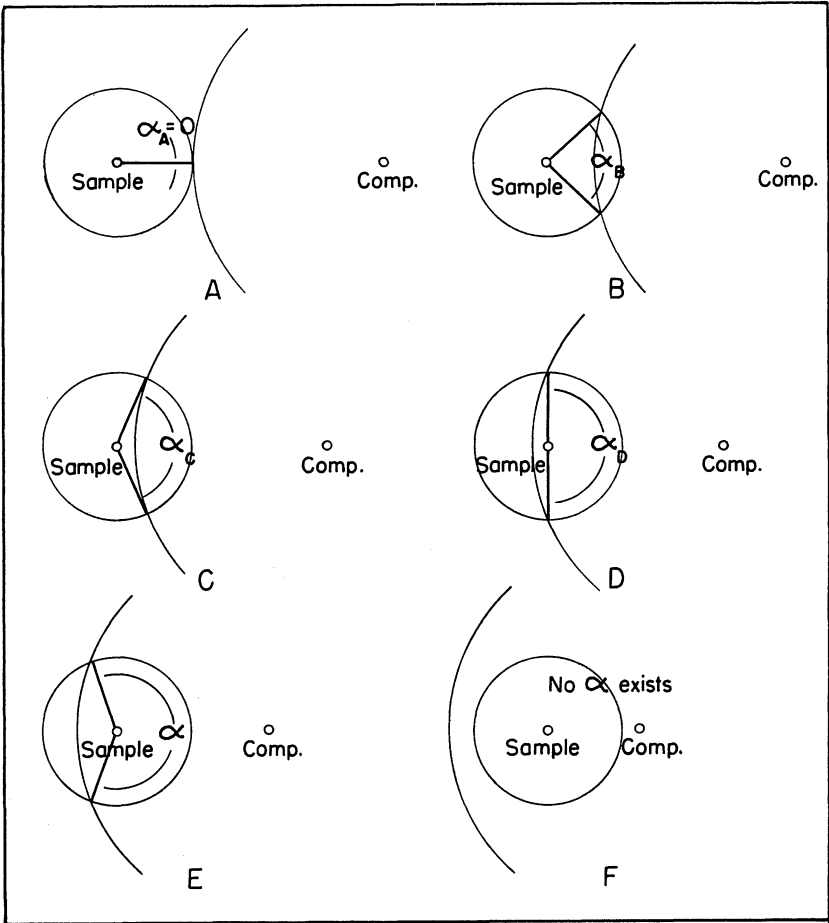


FIG. 9. Effect of size of competitive influence-zone and distance between competing trees on Newnham's competition index when the sample tree is smaller than the competitor.

not be accomplished within the framework of Newnham's procedure. Consequently, his decision probably is as practical a solution to the problem as could be devised.

While working on an individual tree growth study of beech (*Fagus grandifolia* Ehrh.) in Ohio, Fritts (5) developed a measure of point density that involved the overlapping of C.I. Zones. This apparently was done completely independently of Staebler's work. In Fritt's procedure, the C.I.Z. size was governed by the following relationship:

$$A = 2 D \text{.....} (15)$$

where: A = diameter of C.I.Z. circle, in feet;

D = d.b.h.o.b. of tree, in inches.

The source of this relationship was not stated. Fritts cites Rogers (23), who stated that the roots of apple trees growing on sand in Kent, England, spread 2 to 3 times as far as do the branches, while in loam and clay the root spread was about 1.6 times as great as the branch spread. The tie between this and Fritts' relationship is tenuous at best.

To arrive at his competition index, Fritts mapped the sample tree and its competitors, drew in the C.I. Zones on the map, and measured the *overlap areas* within the C.I.Z. of the sample tree with a planimeter, Figure 10 and Appendix A.6. The sum of these overlap areas was divided by the area of the sample tree C.I.Z. to obtain the proportion under competition, then multiplied by 100 to convert to percentage:

$$I_{FG} = \frac{100}{S} \left( \sum_{i=1}^n o_i \right) \text{.....} (16)$$

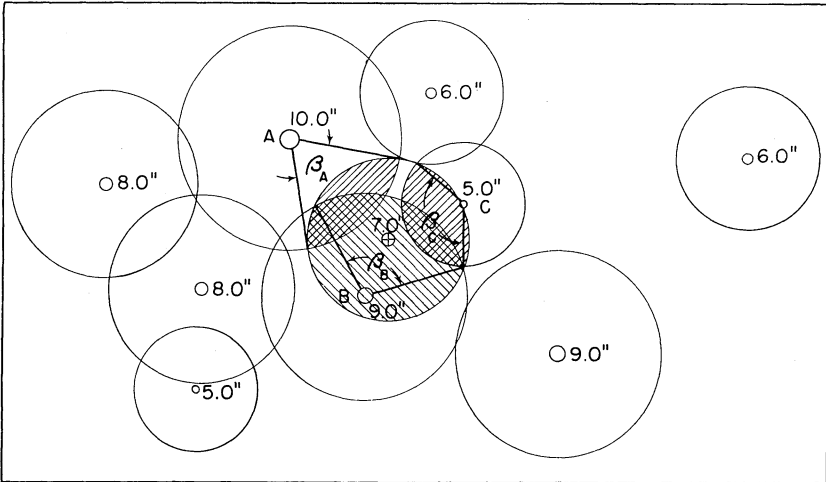
where:  $I_{FG}$  = competition index in per cent of sample tree C.I.Z.;

S = area of the C.I.Z. of the sample tree;

$o_i$  = overlap area of the C.I.Z. of the *i*th competition.

Gerrard (6,7) independently derived essentially the same competition index as the one used by Fritts (Equation 16). However, Gerrard based the size of the C.I.Z. circles on an empirically obtained value for the coefficient b in the equation:

$$R = b D \text{.....} (17)$$



Scale of diameters twice that of locations and C.I. Zones

Radius of C.I.Z., in feet = d.b.h. of tree, in inches,  $\div 1$

$I_{FG} = 124.52\%$

$I_B = 188\%$

$I_O = 86.79$  sq. ft./acre of basal area

FIG. 10. Competitive influence-zone overlap using the Fritts-Gerrard, ( $I_{FG}$ ), the Bella ( $I_B$ ), and the Opie ( $I_O$ ), competition indices. (See Appendices A. 6, A. 7., and A. 8. for computations.)

where:  $R$  = radius of C.I.Z., in feet;

$b$  = radius factor;

$D$  = d.b.h.o.b., in inches.

Gerrard chose an arbitrary sequence of values for  $b$ , then tested the resulting indices for predicting basal area increment. He used the value for  $b$  that resulted in highest correlation between his index and basal area increment. Using data from an area in southern Michigan, he found that the values for  $b$  yielding the best correlations were about 2.25 for red oak (*Quercus borealis* Michx.), 1.75 for black oak (*Q. velutina* Lan.), and 1.25 for hickories (*Carya* spp.) and maples (*Acer* spp.).

In the course of his study, Gerrard compared his competition index with several other indices or methods of expressing point density: Spurr's, a modification of Spurr's, and Newnham's. He found his index to be consistently the most effective for predicting future basal area growth.

Keister (12) accepted the Fritts-Gerrard index in a study he made of point density in plantations of slash and loblolly pine in Louisiana. However, he defined the C.I.Z. in different terms:

$$R = \frac{hl}{m} \text{-----} (18)$$

where: R = radius of C.I.Z.;

h = total height of tree;

m = height to base of live crown;

l = radius of crown at base of live crown, with all in the same units.

His rationale for using this procedure was that the magnitude of the C.I.Z. is not a function of d.b.h. alone but is also influenced by both tree height and length of live crown. If two trees have the same d.b.h., the one that is taller and/or has a deeper live crown should have a competitive advantage over the other. This argument appears sound. Keister, recognizing the difficulty of evaluating the variables in his equation, substituted estimated values for h and l which had been derived from equations with d.b.h. as the independent variable. Therefore, h and l became synthetic variables whose magnitude depended entirely upon d.b.h. and, *per se*, furnished no information.

Like most workers in the field, Keister related his point density index to growth using regression analysis and evaluated the results using correlation coefficients. The equation used in these tests was:

$$y = a + b_1 I_K + b_2 n + b_3 \ln(I_K/n) \text{-----} (19)$$

where: y = d.b.h. growth over growth period;

$I_K$  = index of competition;

n = number of trees whose C.I. Zones overlapped that of sample tree.

$\ln(I_K/n)$  = natural logarithm of  $I_K/n$

The  $I_K/n$  term was included to account for the fact that any given degree of overlap with several competitors has less impact on the growth of a sample tree than the same amount of overlap coming from a single competitor. In most cases, this second index

proved to be more effective than the first in reducing the residual sum of squares.

Bella (1,2) also developed a competition index based on the ratio of the sum of the overlap areas of the C.I. Zones to the area of the C.I.Z. of the sample tree, Figure 10 and Appendix A.7. However, he apparently borrowed a couple of ideas from Newnham (20) and used them while developing a modification of the Fritts-Gerrard procedure.

The magnitudes of the C.I. Zones in the Bella method are based on the relationship between the crown diameters of open-grown trees and their d.b.h.o.b.'s which must be established empirically for each species considered. Bella, like Newnham, recognizing that the C.I.Z. is not necessarily coincident with the extent of open-grown trees of a given d.b.h., applied a correction factor (K) to the predicted crown diameters:

$$A = P K \text{-----} (20)$$

where: A = diameter of C.I.Z.;

P = predicted crown diameter;

K = correction factor.

Bella believed that the magnitude of K would be dependent on species and, probably, age and site as well. Using empirical methods, he found that a K of approximately 3.00 worked well with aspen (*Populus* spp.), while a value between 2.7 and 3.2 seemed appropriate for jack pine (*Pinus banksiana* Lamb.) and Douglas fir, and 1.5 for red pine (*Pinus resinosa* Ait.).

Bella, again like Newnham, recognized that a given per cent overlap of C.I. Zones is not a fully satisfactory measure of inter-tree competition since it does not take into account the relative sizes of the trees. To overcome this deficiency, Bella used essentially the same method used by Newnham. He multiplied the per cent overlap for each competitor by the ratio of the d.b.h.o.b. of the competitor to the d.b.h.o.b. of the sample tree. This weights the per cent overlap according to the size of the competitor. Bella, however, did not stop here. He reasoned that the tree size effect would differ by species and, perhaps, for other reasons. Consequently, he raised the ratio of diameters to a variable power (x), which he evolved empirically. He estimated the value of x to be approximately 2.0 for aspen, jack pine, and red pine, while for Douglas fir it should be about 1.2.

Expressed mathematically, Bella's competition index takes the form:

$$I_B = \frac{1}{S} \sum_{i=1}^n [(O_i) (D_i/D_s)^x] \dots\dots\dots (21)$$

where:  $I_B$  = index of competition;

$S$  = area of C.I.Z. of sample tree;

$O_i$  = overlap area of the C.I.Z. of the  $i$ th competitor;

$D_i$  = d.b.h.o.b. of  $i$ th competitor;

$D_s$  = d.b.h.o.b. of sample tree;

$X$  = variable power.

Bella compared the results obtained from his index with those from both the Fritts-Gerrard and the Opie (22) indices. He found that his procedure yielded a significantly better estimate of growth than either of the others.

### Miscellaneous Methods

Steneker and Jarvis (28), when studying the effect of competitive pressure on individual white spruce trees, used a series of competition indices that involved the trees on a small plot of a fixed radius (25 feet), centered on the sample tree. One of these, the sum of the basal areas of the competing trees, has already been mentioned. The remaining indices were:

$$I_{SJ1} = \sum_{i=1}^n (D_s/G_i) \dots\dots\dots (22)$$

$$I_{SJ2} = \sum_{i=1}^n (D_s/G_i^2) \dots\dots\dots (23)$$

$$I_{SJ3} = \sum_{i=1}^n (D_s^2/G_i) \dots\dots\dots (24)$$

$$I_{SJ4} = \sum_{i=1}^n (D_s^2/G_i^2) \dots\dots\dots (25)$$

where:  $I_{SJ1}$  through  $I_{SJ4}$  = indices of competition;

$n$  = number of trees in sample plot;

$D_s$  = d.b.h. of sample tree, in inches;



$G_i$  = distance from sample tree to  $i$ th competitor, in feet.

The sample tree itself was never allowed to contribute anything to these measures of competitive pressure. This appears logical since the pressure *on* the sample tree was being evaluated. Figure 1 and Appendix A.1. show examples of these indices.

Opie (22) developed an index of competition that draws on both the overlapping C.I.Z. concept and the basal area per unit land area concept. The sizes of the C.I. Zones were defined in terms of d.b.h., with a separate multiplier ( $m$ ) for each of three site classes:

$$R = m D \dots\dots\dots (26)$$

Opie tested his procedure using data from stands of *Eucalyptus* spp. in Australia. For the best sites  $m$  was estimated to be 1.20, for medium sites it was set at 1.35, and for poor sites  $m$  was set at 1.45. These multipliers actually represent the radius of the C.I.Z. in feet per inch of d.b.h.o.b. Thus, the BAF (from the Bitterlich method) associated with each of the multipliers can be determined from the relationship expressed in equations (1), (29), and (30). With these multipliers, the corresponding BAF values are 52, 41, and 36 square feet per acre. Using an angle gauge with a BAF appropriate to the site, the C.I.Z. of the sample tree is determined, then the C.I. Zones of the competing trees are similarly derived. The index of competition, called by Opie the "Zone count," is computed as follows:

$$I_o = \frac{\text{BAF}}{S} \sum_{i=1}^n O_i \dots\dots\dots (27)$$

where:  $I_o$  = index of competition;

BAF = basal area factor;

$S$  = area of C.I.Z. of sample tree;

$O_i$  = overlap of the  $i$ th competition.

In essence, the total amount of overlap is related to the area of the sample tree's C.I.Z. and the resulting quotient is considered to be the equivalent of an angle count. Thus, the angle count multiplied by the BAF yields an estimate of the basal area per unit land area centered on the sample tree.

This procedure weights the effects of the competitors on a basis

which recognizes differences in tree size as well as differences in distance from the sample tree.

Opie recognized that there would be practical difficulties in using the aforementioned procedure. As a result, he developed a field procedure which would yield estimates of values obtained by the formal procedure. Discussion of this field procedure is omitted here.

Like Gerrard, Opie compared the effectiveness of his procedure with several others (fixed-radius plots, variable-radius plot proportional to tree d.b.h., Bitterlich's and Spurr's). The response variable used in these tests was basal area increment. The results indicated that Opie's method yielded results which were similar to the others.

Latham (16) has proposed a competition index that is unusual in that it requires the use of stereoscopic pairs of large scale, vertical aerial photographs. A stereogram of an inverted cone is constructed on a transparent base. The image of the sample tree is viewed stereoscopically and the stereogram of the cone is superimposed on the stereopair in such a manner that the apex (bottom point) of the cone is at the foot of the sample tree. Trees whose crowns penetrate the cone are considered competitors. Latham did not elaborate on how the competition would be expressed beyond stating that the cone was acting as a vertically oriented angle gauge and made reference to Bitterlich's angle-count theory. The trees whose crowns penetrate the cone are *in* trees. In this case, the trees are sampled with probability proportional to height and the count of *in* trees has no direct connection with basal area. It is possible that Latham intended the simple count to be the measure of competitive pressure but, since he apparently was more concerned with the photogrammetric than the silvical and mensurational aspects of the problem, he left the latter unresolved. This approach is intriguing and someday might be developed to serve as the basis for a procedure that would be useful to foresters who use aerial photographs.

### SOURCE OF DATA

To evaluate the effectiveness of a point density expression as a measure of competitive pressure, it is necessary to have data from a stand or stands of trees which have been measured periodically over a reasonable span of time. With such data, conditions found at the *beginning* of a growth period can be related to *subsequent*

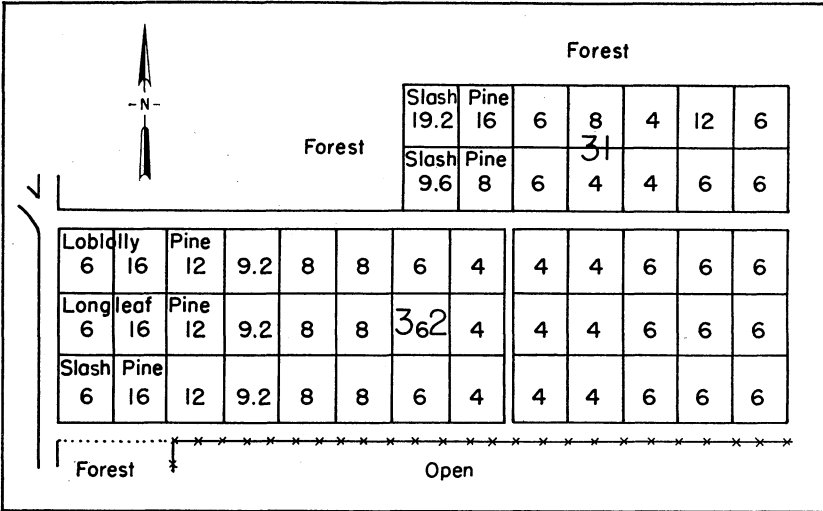


FIG. 11. Map of experimental area. Numbers in plots indicate spacing in feet.

growth. Data of this type were available for this study from plantations of loblolly, longleaf, and slash pines established at Auburn, Alabama, for a spacing and thinning study by the Agricultural Experiment Station of Auburn University.<sup>3</sup>

The layout of these plantings is shown in Figure 11. Essentially, the individual plantation units were rectangular, 1/4-acre plots grouped into two blocks, Block 31 on the north of the transverse road and Block 32 on the south side of the road. No statistical design formed the basis for the assignment of species or spacings to the plots. Since slash pine apparently was of more interest to the investigators than were the other two species, it was planted most extensively and occupies all the plots in Block 31 and one-third of the plots in Block 32.

The original spacings of the trees varied from 4×4 feet to 19×19 feet, with 6×6 feet used more than any other spacing. The trees were precisely located within the plots. Those who conducted the planting were guided by wires stretched across the plots. This made it possible in this study to indicate tree position within a matrix and to use row number, column number, and original spacing to compute distances between trees.

The plantations were established in 1932 with 1-year-old seed-

<sup>3</sup> More detailed information about these plantations may be obtained from Livingston (19).

lings grown in the University's small, temporary tree nursery. Thinnings were made several times in subsequent years. The thinning policy apparently was uniform for all plots. It consisted primarily of thinning from below and salvaging trees that probably would otherwise have been naturally lost. Prior to each thinning, following the marking, a complete cut and leave inventory was made of all the plots. Usable data were obtained from the last four of these inventories (in 1945, 1950, 1955, and 1962) for slash pine and from the last three inventories for loblolly and longleaf pine.

The primary data obtained in the course of these inventories were d.b.h.o.b. of all living trees, which were measured to the nearest 1/10 inch. In addition, crown class was recorded. Site index was estimated for each individual plot when the trees were 31 years old, using the site index curves from *Volume, yield, and stand tables for second-growth southern pines* (29).

Four sample trees were chosen subjectively from each of the plots. Where possible, one of these was from each of the four crown classes: dominant, codominant, intermediate, and overtopped.<sup>4</sup> These sample trees were chosen from trees that had survived to age 31 and were located deep enough within the plots so that the trees with which they were competing were plot companions. Consequently, no sample trees were chosen in the outer 7 rows in plots with 4×4 feet spacing, in the outer 4 rows in plots with 6×6 foot or 8×8 foot spacing, or in the outer 2 rows in plots with spacings greater than 8×8 feet. In some cases, not all crown classes were represented among the available sample trees. In such cases the deficits were made up by arbitrarily choosing substitutes. It was felt that subjective sampling would be acceptable since, at the time of selection, no knowledge was available concerning either the growth or the point density.

Preliminary analyses of the data indicated that the slash pine in Block 31 responded in a much different way to point density than did the slash pine in Block 32. Therefore, the two sets of slash pine data were kept separate and were analyzed independently of one another.

<sup>4</sup> In block 32 sample trees were chosen from among the overtopped trees. This was not done in Block 31. The preliminary work was done in Block 31 and in this phase it was thought that the growth response of overtopped pines to point density would be negligible and of little importance. Consequently overtopped trees were not used as sample trees. Later this opinion was changed but the decision was made to continue to use the original samples from Block 31 so as not to lose the time and effort invested in the analyses of those samples.

## STATISTICAL DESIGN

Initially, the study plan called for the development of mathematical models that could be used to predict periodic annual increment in both d.b.h.o.b. and tree basal area. These models were to include, as independent variables, site index and age, d.b.h.o.b., crown class, and point density at the beginning of the period. The reduction in residual sum of squares attributable to the point density expression would be used as the measure of the expression's effectiveness, or power.

A basic, theoretical model using these variables was devised but could not be fitted to the data by conventional regression procedures because of nonlinearity in the coefficients. Though iterative fitting procedures could have been used, the sheer magnitude of the required computations, even with the aid of a large computer, caused this approach to be rejected.

An attempt was made to develop models amenable to linear regression fitting procedures. When this was done, however, the point density expressions often were eliminated in the fitting process and did not appear in the final equations. In the cases of one of the slash pine and one of the loblolly pine datasets not one of the point density expressions was retained. Consequently, the regression approach to evaluation of the effectiveness of the various point density expressions was abandoned.

As a result of these experiences, it was decided that the degree of relationship between the growth and point density values would be measured in terms of simple correlation coefficients. It was further decided to retain the two original tree growth variables (periodic annual increment in tree basal area and periodic annual increment in d.b.h.o.b.) and to test the point density expressions with each of these two variables. The point density expressions that were tested are listed in the next section.

Statistical significance of the correlation coefficients themselves were determined by standard procedures (24). However, since procedures for making multiple comparisons among correlation coefficients are not known, nothing could be done to determine the statistical significance of the differences between the large numbers of correlation coefficients generated in the course of the study. Individuals making use of the tables in this report will be obliged to draw their own conclusions with respect to the differences among coefficients.

The relationship between growth of a sample tree and a meas-

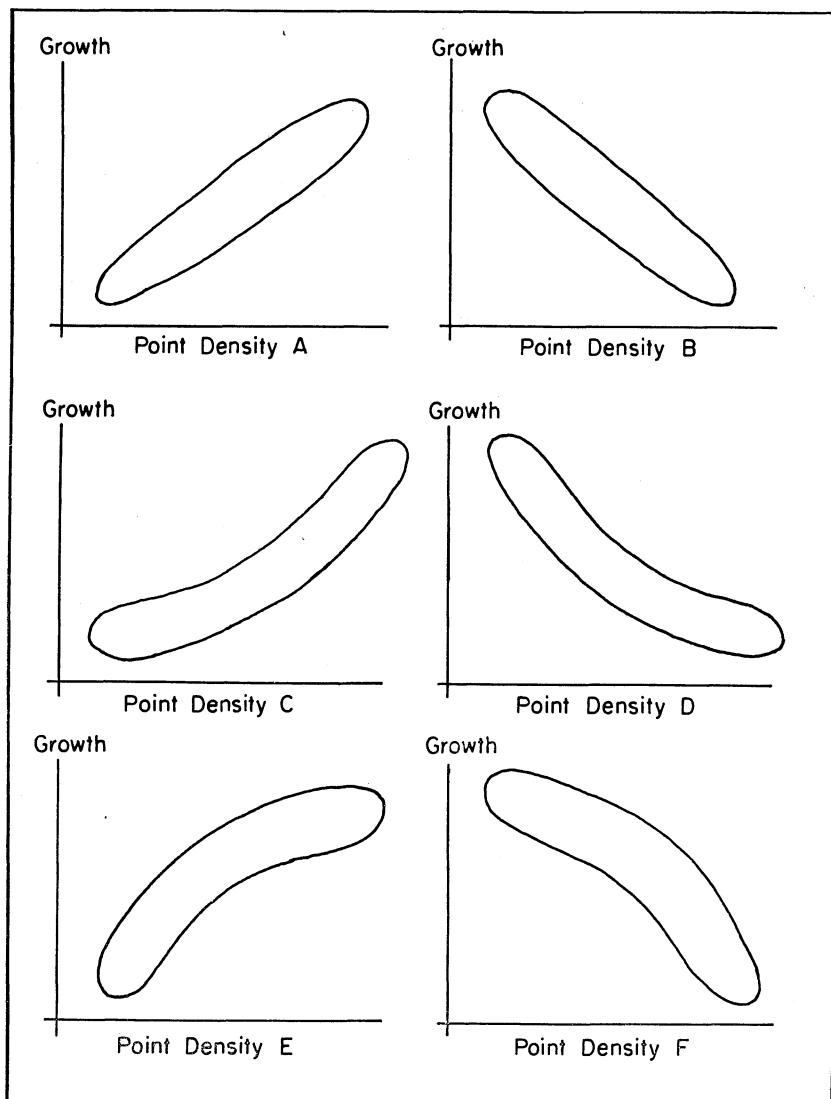


FIG. 12. Possible relationships between sample tree growth and point density.

ure of competitive pressure against that tree may take on several conceivable forms, as shown in Figure 12. The correlation can be either positive or negative, depending on the nature of the point density expression. Furthermore, the relationship has a shape, which may be linear, as in Figure 12 A and B, or curvilinear, as

in Figure 12 C, D, E, and F. Though the shape could be even more complex, that possibility was ignored in this study.

To recognize possible curvilinearity, three coefficients were computed for each growth variable-point density expression-species-crown class-level<sup>5</sup> combination. With the first correlation coefficient, the relationship was assumed to be linear and no modifications were made to either the growth or the point density values. In the second, square roots of the point density values were used. This anamorphosis of the point density scale would tend to linearize the relationship in the event that it was of the Figure 12, C or D type. In the third computation, point density values were squared, with the effect that the relationship would tend to linearity if it were of the Figure 12, E or F type. It was assumed that the highest correlation coefficient would be associated with the procedure that most nearly linearized the relationship.

### POINT DENSITY EXPRESSIONS TESTED

The following point density expressions were tested:

(1). Basal area per acre from fixed radius plots centered on the sample tree. Two plot sizes were used, the first with a radius of 14.42 feet (0.015 acre) and the second with a radius of 26.33 feet (0.050 acre).

(2-5). Steneker and Jarvis' expressions  $I_{SJ1}$ ,  $I_{SJ2}$ ,  $I_{SJ3}$ , and  $I_{SJ4}$  (Equations 22, 23, 24, and 25 respectively), in conjunction with the plots described above.

(6). Bitterlich's angle-count, with basal area factors of 5, 10, 15, 20, 25, 30, and 40 square feet per acre.

(7). Spurr's angle-summation, using the first 4, 6, 8, 10, 12, 14, and 16 trees subtending the largest angles.

(8). Brown's growing space method.

(9). A modification of Brown's method in which the relative sizes of the trees at the ends of each of the lines were taken into account. Following is a description of the algorithm used. (Also see Figure 11.)

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<sup>5</sup> Level refers to the magnitude of a controlling variable, within a given point density expression, where that magnitude is arbitrarily assigned by the investigator. For example, in the Bitterlich method the person doing the work may decide to use several different BAF's. Each of these represents a level. Again, in the Spurr method, the number of competing trees that are to be considered can be controlled by the investigator. Each such number is a level. Many of the point density expressions have variables of this type.

- (a). A map was constructed for each sample tree as in Figure 5.
- (b). The sample tree was connected to each of its competitors with a straight line.
- (c). The distance ( $T_i$ ) between the sample tree and the  $i$ th competitor was measured.
- (d). The diameters d.b.h.o.b.) of the two trees were averaged:  

$$\bar{D} = (D_s + D_i)/2$$
- (e). The difference between the larger of the two diameters ( $D$ ) and  $\bar{D}$  was computed:  

$$f = D - \bar{D}$$
- (f). The value  $q$  was computed:  

$$q = \frac{fT_i}{2D} + \frac{T_i}{2}$$
- (g). The distance  $q$  was laid off on the line connecting the two trees using the *larger* tree as the origin.
- (h). Perpendiculars were constructed through the points located in this manner, polygons were developed, and their areas were measured as in the Brown method.

(10-15). Staebler's indices  $I_{S1}/F$ ,  $I_{S2}/F$ ,  $I_{S3}/F$ , and  $I_{S4}/F$  (See equations 6, 7, 8, 9, and 10). In addition, index  $I_{S1}$  was used in its uncorrected form. Furthermore, a modification of the latter was used in which  $I_{S1}$  was expressed as a percentage of the diameter of the sample tree's C.I.Z. This modification was labelled  $I_{S5}$ , see Figure 6 and Appendix A.4.

(16-17). Newnham's index  $I_N$  (see Equation 12). In addition, a modification of this index was used where the correction factor  $A_i/A_s$  was deleted. This was labelled  $I_{N2}$ .

(18). The Fritts-Gerrard index  $I_{FG}$  (see Equation 16).

(19). Bella's index  $I_B$  (see Equation 16).

(20). Opie's index  $I_o$  (see Equation 27).

(21). In addition, to provide a standard of comparison, conventional basal area per acre on an entire quarter-acre plot, not necessarily centered on the sample tree (SBA) was tested.

In the case of the fixed plot, the Bitterlich, and the Spurr methods, separate basal areas per acre were computed with the sample tree both included and excluded.

In the case of the methods based on the concept of the C.I.Z., a common definition of the C.I.Z. radius was used rather than



each of those used by the individual investigators. Basically, the common relationship used was:<sup>6</sup>

$$R = 1 + D \text{-----} (28)$$

where: R = radius of the C.I.Z., in feet;

D = d.b.h.o.b., in inches.

The derived radius, R, was appropriate for open-grown trees. Since both Newnham and Bella recognized that the C.I.Z. of a tree in a closed stand probably would be different from one in an open situation, they introduced correction factors into their C.I.Z. computations to compensate for this difference. Both investigators based their correction factors on empirical evidence. This policy was followed in this study. The *effective* C.I.Z. radius was defined as:

$$R = (1 + D) K \text{-----} (29)$$

where: R = effective C.I.Z. radius, in feet;

D = d.b.h.o.b., in inches;

K = correction factor.

Values for K were arbitrarily set at 1.0, 1.3, 1.6, 1.9, 2.2, 2.5, 2.8. Note that when K = 1.0, the result is the same as when no correction value is used.

As was previously described, the angles  $\alpha_1$  used by Newnham as a measure of competitive pressure, Equation 12 and Figures 8 and 9, usually are consistent with theoretical considerations, though in some cases the angle decreases as the competitive pressure increases. Newnham recognized this and, in general, the procedures used in this study were in agreement with those he used. However, some changes were made. Whenever the situations shown in Figures 8E and 8F existed, the value assigned  $\alpha_1$  was set equal to that in Figure 8D, where  $\alpha$  reaches a maximum for the given pair of circles. In contrast, Newnham assigned  $\alpha$  a value of zero under these conditions. With Newnham's procedure, the measure of competitive pressure under certain situa-

<sup>6</sup>D.b.h.o.b. and crown radius data were obtained in the Auburn area from 34 trees (24 loblolly, 5 longleaf, and 5 shortleaf pines) that apparently had always been open-grown. The diameter range was from 3.7 to 28.1 inches and the diameters were well distributed within this range. Apparently there were no appreciable species differences. The resulting equation was:  $R = 1.1 + 0.93 D$ . The correlation coefficient was 0.966. Thus, the relationship used (Equation 28) seemed to be reasonable.

tions can decrease as the actual pressure increases. Since the procedure used in this study prevented that possibility, it appeared to be more logical.

In the case of Bella's method, a series of exponents ( $x$ ) were used with the weighting factor  $(D_i/D_s)^x$ , as shown in Equation 21. The exponents tested were 0.5, 0.8, 1.0, 1.1, 1.4, 1.7, 2.0, 2.3, 2.6, 2.9, and 3.0.

The basal area factor (BAF) used in Opie's method varied according to the correction factor (K) in Equation 30<sup>7</sup>:

$$\text{BAF} = 43560 / (25 K)^2 \quad (30)$$

Consequently, the BAF's used were: 69.70, 41.24, 27.22, 19.31, 14.40, 11.15, and 8.89.

## RESULTS AND DISCUSSION

After computation of the correlation coefficients, the printed output of the computer was searched for the highest correlation coefficient associated with each combination of growth variable, point-density expression, species, crown class, and level, regardless of the shape of the relationship. These correlation coefficient maxima were then plotted against crown class in the case of the point density expressions where level was not a factor, or against level by crown class in the case of the expressions where level was involved, Figures 14-47. From these Figures, one can obtain an idea of the effect of crown class and level differences on the efficiency of the several point density expressions.

Each of the aforementioned maximum correlation coefficients was a member of a three-member set, or triplet, which showed the results using the original data and the two transformations of that data. These sets or triplets of correlation coefficients are shown in Appendix B, Tables 1-24. The statistical significance of

<sup>7</sup> Refer to Equations 1 and 29.

$$\text{Equation 1: } \text{BAF} = \frac{Cr^2}{R^2}$$

$$\begin{aligned} \text{Equation 29: } R &= (1 + D) K \\ &\text{when } D = \text{d.b.h.o.b.} = 24 \text{ inches;} \\ R &= (1 + 24) K = 25K. \end{aligned}$$

When  $D = 24$  inches,  $r = 12$  inches or 1 foot.

Substituting  $C = 43560$

$r = 1$ ; and  $R = 25K$  in Equation 1 yields:

$$\text{BAF} = \frac{43560(1)^2}{(25K)^2} = 43560 / (25K)^2$$

each maximum is shown adjacent to that maximum but has nothing to do with the other two members of the set. If one desires to find the significance of the latter two, he should use standard procedures (24). These tables also show which were the best levels in cases where levels were involved.

This type of study does not lend itself well to a statistical analysis. Procedures have not yet been developed for making multiple comparisons of correlation coefficients. Furthermore, any procedure making use of regression analysis would be impractically massive. Therefore, the reader should keep in mind that most of the following discussion was based on the author's subjective judgment and reasoned interpretations of the results of this study.

### Datasets

The most striking thing encountered in the course of the study was the behavior of the different groups of data. As might be expected, there were species differences, but the greatest difference, across the board, involved the two sets of slash pine data. The slash pine results from Block 31 (see the map in Figure 11 for relative location) are what one might expect. Point density was correlated significantly with growth in most cases and the results were consistent with theoretical considerations. In other words, when a point density expression indicated increasing competitive pressure the growth rate slowed. The slash pine results from Block 31, however, were highly erratic and the relationship between growth and the point density values were usually weak or non-existent. Often the growth rate increased as the point density increased, which, superficially anyway, is not logical.

The reason for this divergence of behavior between the two blocks of slash pine is not clear. Both blocks were planted at the same time and, presumably, the genetic backgrounds of the trees were generally similar. Though there was a statistically significant difference between the mean site indices, the magnitude of the difference was not great.<sup>8</sup> Furthermore, it is difficult to visualize the mechanism that would cause differences in site quality to have so much effect on the relationship between individual tree growth and point density. Both blocks had been thinned, and the thinning regime, rationale, and schedule apparently were the same for both blocks.

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<sup>8</sup> Mean site indices were: Block 31, 91.1; Block 32, 85.8. The difference was significant at the 0.01 level. The variances were homogeneous.

The two blocks, however, had different surroundings. As can be seen in Figure 11, the slash pine plots in Block 32 were bordered on the south by open fields and on the north by the longleaf pine plots. Since the longleaf pine stand was slow to develop, the slash pine stand, during much of its life, was essentially open on both sides. In Block 31, on the other hand, the slash pine plots were bordered on the south by the loblolly pine plots of Block 32 and on the north either by other planted pine stands of approximately the same age or by natural timber. Consequently, Block 31 more nearly represented closed forest condition while the slash pine in Block 32 probably had been influenced by openness on both sides of the single row of plots and represented an exaggerated edge situation, where substantially more light was available. With more light generally available the effect of point density conceivably could be greatly modified.

This thesis receives added support when one examines the data from the longleaf and loblolly pine plots. Longleaf pine behaved as one would expect and was much like the Block 31 slash pine. It developed under conditions of competition for sunlight, and possibly for moisture, from the taller stands of loblolly and slash pine which bordered it to the north and south. Since the loblolly pine, which behaved somewhat more erratically than the Block 31 slash pine or the longleaf pine, was bordered on one side by the slow-to-develop longleaf for a substantial portion of its life, it had been reasonably free of competition for sunlight. Consequently, the edge effect probably influenced the loblolly pine results.

These findings suggest that the effect of proximity to the edge of a stand may extend deeper into a stand than is generally recognized. Research workers involved with responses of individual trees to treatments of various kinds should be aware of this possibility and should locate their plots so that the treatment effects will not be confounded with edge effect.

### **Crown Classes**

The behavior of the crown classes can best be visualized by a study of the graphs in Appendix B, Figures 14-47. In general, estimated point density had the greatest effect on growth in the case of the lower crown classes and, probably, the intermediate class showed the most consistent results. Only in the case of the slash pine in Block 31 did the dominants show consistently high cor-

relations. Within the other groups the correlations for the dominant and codominant classes fluctuated widely and erratically. These results are not necessarily illogical. The dominants and codominants had been able to hold their positions in the canopy partly because they were more aggressive, while the intermediate and overtopped trees slipped to their positions because they were relatively susceptible to competition. The extremely erratic nature of the results from the dominant and codominant classes in both the slash pine of Block 32 and the loblolly pine probably was largely due to the edge effect previously discussed. The lower crown classes acted in a much more expectable manner than did the upper classes in these stands.

The overtopped trees showed a weaker relationship between growth and point density than did the intermediates. This probably was due to the very small magnitudes of the growth increments. The d.b.h.'s were measured to 1/10 inch. It is possible that these measurements were too coarse, resulting in many trees showing the same increment over the period when actually there was a differential response to competitive pressure which could only be detected with measurements using units smaller than 1/10 inch.

For a few of the point density expressions, the sign of the correlation coefficient for ALL trees was opposite to the correlation coefficients for each of the individual crown classes. This is demonstrated in Figure 13. Regression lines are shown since they are easier to comprehend than scattergrams. As can be seen, the individual crown class curves slope down toward the right while the overall curve slopes upward. When this occurs it indicates that the point density expression detects the fact that large trees (e.g., dominants), in general, regardless of competitive pressure, grow more than smaller trees (e.g., intermediates or overtopped). However, within a crown class, growth falls off as competitive pressure increases. This differential in growth response made it necessary to recognize crown class in this study.

### Method of Expressing Growth

In general, the relationship between basal area growth and point density was similar to that between d.b.h. growth and point density. However, in the majority of cases, the correlation was somewhat better for the basal area data than for the d.b.h. data. No pattern emerged to support an argument that, under given cir-

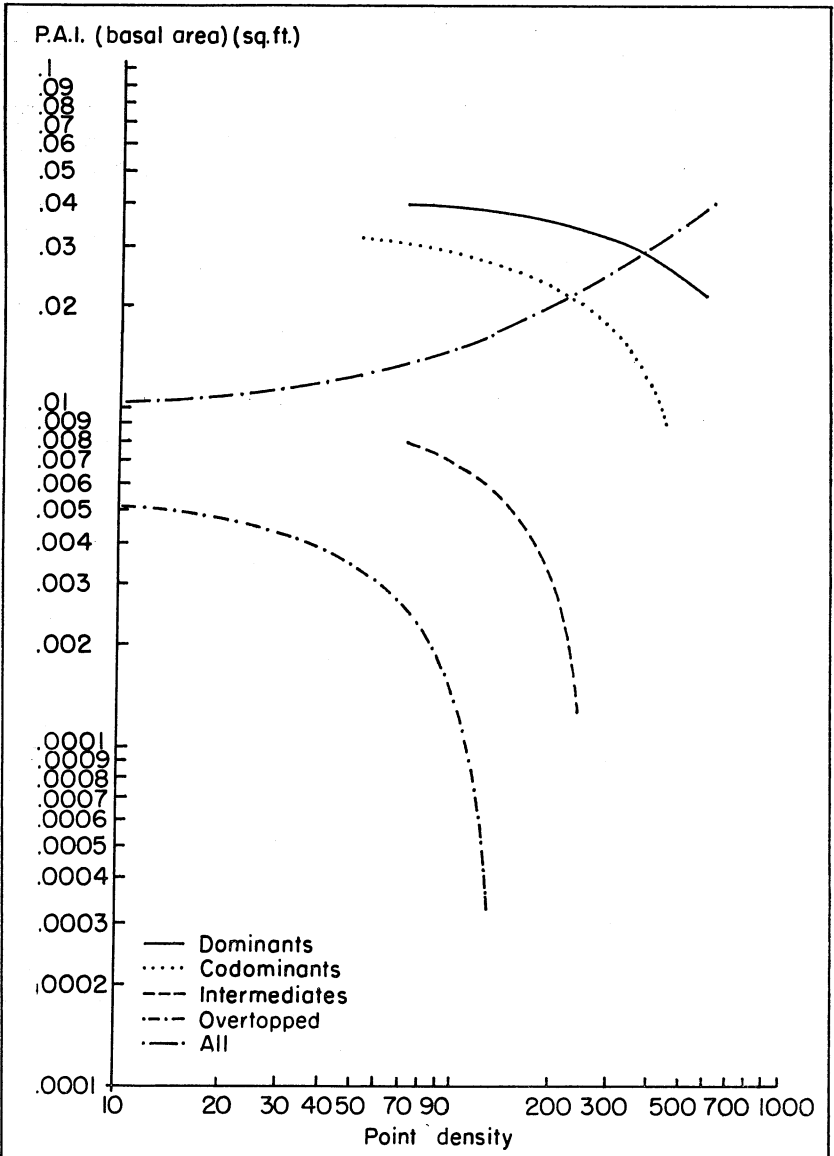


FIG. 13. Relationship between periodic annual increment in basal area and the point density values obtained using Staebler's competition index  $I_{S1}$  with longleaf pine.

cumstances, one of the growth variables would correlate better than the other with a selected point density expression.

Theoretically, the basal area growth-point density relationship should be of more interest or utility than the d.b.h. growth-point density relationship because basal area partially accounts for size of the tree. For example, any unit of d.b.h. growth on a small tree is the same as that same unit of d.b.h. growth on a large tree as far as d.b.h. growth is concerned. When the identical units d.b.h. increments are converted into basal area increments, however, the larger tree exhibits more basal area growth than does the small tree. The somewhat higher correlation encountered when basal area growth was the dependent variable may result from the partial accounting for tree size.

### Point-Density Expressions

Examination of the correlation coefficients in the tables and figures of Appendix B will reveal that, in general, the relationships between individual tree d.b.h. or basal area growth and point density were weak. The correlation coefficients ranged in magnitude from a high of 0.8448 (Bella's index  $I_B$ , basal area growth, dominant trees, loblolly pine) to a low that was essentially zero. Most of the correlation coefficients were in the range from 0.3 to 0.6. Since a substantial proportion of the correlations were not significant (0.05 level of probability), serious doubts of the utility of the point density expressions arise.

Tables 1 and 2 show the rankings of the 24 different point density expressions tested using the results from the basal area growth of the slash pine in Block 31 and the longleaf pine in Block 32. These two sets of results were chosen because they represent the conditions under which the point density expressions appeared to be most effective. Rankings for the other groups can be developed from the information in Appendix B, Tables 1-24.

As can be seen, no single point density expression is clearly superior to the others. However, one can generalize to the extent that the expressions based on C.I.Z. overlaps usually ranked high and that the Steneker-Jarvis and the Brown expressions usually ranked low. In addition, stand basal area was about as reliable a predictor of individual tree growth as any point density expression tested. Furthermore, stand basal area ranked highest with the dominants.

That the Fritts-Gerrard and Opie indices ( $I_{FG}$  and  $I_o$ ) yield almost exactly the same result appears logical because, in essence

TABLE 1. RANKING OF POINT DENSITY EXPRESSIONS WHEN THE GROWTH EXPRESSION WAS PERIODIC ANNUAL INCREMENT IN BASAL AREA, SLASH PINE, BLOCK 31<sup>1</sup>

Rank	Crown Classes									
	Dominant		Codominant		Intermediate		Overtopped		All	
	Expr.	r	Expr.	r	Expr.	r	Expr.	r	Expr.	r
1	SBA	-.8146	I <sub>B</sub>	-.6123	SBA	-.5860			I <sub>B</sub>	-.7550
2	I <sub>N</sub>	-.8105	I <sub>N</sub>	-.6028	I <sub>S4</sub> /F	-.5733			I <sub>N</sub>	-.7337
3	I <sub>B</sub>	-.8066	I <sub>FG</sub>	-.5176	Spurr w/o	-.5621			I <sub>FG</sub>	-.6205
4	I <sub>FG</sub>	-.8041	I <sub>O</sub>	-.5174	I <sub>B</sub>	-.5612			I <sub>O</sub>	-.6188
5	I <sub>O</sub>	-.7924	I <sub>N2</sub>	-.5125	I <sub>N</sub>	-.5476			I <sub>N2</sub>	-.6094
6	I <sub>N2</sub>	-.7886	Fixed w/o	-.4914	Fixed w/	-.5462			Fixed w/o	-.5853
7	Bitt. w/o	-.7841	Bitt. w/o	-.4816	I <sub>FG</sub>	-.5449			Brown mod.	+.5655
8	I <sub>S5</sub>	-.7774	Bitt. w/	-.4796	I <sub>O</sub>	-.5449			Bitt. w/o	-.5541
9	I <sub>S3</sub> /F	-.7728	Fixed w/	-.4548	Fixed w/o	-.5439			Bitt. w/	-.5511
10	I <sub>S4</sub> /F	-.7665	I <sub>S5</sub>	-.4509	Spurr w/	-.5436			SBA	-.5481
11	I <sub>S1</sub>	-.7665	I <sub>S3</sub> /F	-.4379	I <sub>N2</sub>	-.5420			Fixed w/	-.5391
12	I <sub>S2</sub> /F	-.7565	I <sub>S1</sub> /F	-.4229	I <sub>S3</sub> /F	-.5413			I <sub>S3</sub> /F	-.5380
13	Fixed w/o	-.7527	Brown mod.	+.3810	I <sub>S1</sub>	-.5262			Spurr w/o	-.5344
14	Fixed w/	-.7431	I <sub>S1</sub>	-.3475	I <sub>S5</sub>	-.5221			I <sub>S5</sub>	-.5262
15	Spurr w/	-.7391	Brown	+.3430	I <sub>S2</sub> /F	-.5123			Spurr w/	-.5231
16	Spurr w/o	-.7336	SBA	-.3423	Bitt. w/	-.4724			I <sub>S1</sub> /F	-.4995
17	I <sub>S1</sub> /F	-.7327	I <sub>SJ1</sub>	-.3013	Bitt. w/o	-.4705			Brown	+.4952
18	I <sub>SJ2</sub>	-.7158	I <sub>S1</sub>	-.2874	I <sub>S1</sub> /F	-.4659			I <sub>S2</sub> /F	-.4338
19	I <sub>SJ1</sub>	-.7147	Spurr w/o	-.2672	I <sub>SJ3</sub>	-.4568			I <sub>SJ2</sub>	-.4284
20	I <sub>SJ4</sub>	-.7118	I <sub>S4</sub> /F	-.2587	I <sub>SJ4</sub>	-.4377			I <sub>SJ1</sub>	-.4122
21	I <sub>SJ3</sub>	-.6914	I <sub>SJ2</sub>	-.2569	I <sub>SJ1</sub>	-.4294			I <sub>S4</sub> /F	-.3861
22	Brown mod.	+.6765	Spurr w/	-.2346	I <sub>SJ2</sub>	-.4154			I <sub>S1</sub>	-.3192
23	Brown	+.6481	I <sub>SJ3</sub>	-.1604	Brown	+.2736			I <sub>SJ4</sub>	-.3066
24	Bitt. w/	-.6409	I <sub>SJ4</sub>	-.1528	Brown mod.	+.2123			I <sub>SJ3</sub>	-.2628

<sup>1</sup>There were no overtopped trees in this set. For an explanation see Footnote 4.



TABLE 2. RANKING OF POINT DENSITY EXPRESSIONS WHEN THE GROWTH EXPRESSION WAS PERIODIC ANNUAL INCREMENT IN BASAL AREA, LONGLEAF PINE

Rank	Crown Classes									
	Dominant		Codominant		Intermediate		Overtopped		All	
	Expr.	r	Expr.	r	Expr.	r	Expr.	r	Expr.	r
1	SBA	-.5492	Spurr w/o	-.7930	Spurr w/	-.7828	Bitt. w/	-.6516	I <sub>N</sub>	-.8027
2	Bitt. w/o	-.4898	Spurr w/	-.7924	Spurr w/o	-.7606	Bitt. w/o	-.6487	I <sub>B</sub>	-.7941
3	I <sub>S3</sub> /F	-.4485	I <sub>SJ1</sub>	-.7214	I <sub>N</sub>	-.6917	I <sub>FG</sub>	-.6340	Brown mod.	+ .7455
4	I <sub>N2</sub>	-.4466	I <sub>SJ3</sub>	-.7134	I <sub>N2</sub>	-.6715	I <sub>O</sub>	-.6340	I <sub>N2</sub>	-.6259
5	I <sub>S1</sub>	-.4427	I <sub>S2</sub> /F	-.7119	I <sub>S4</sub> /F	-.6676	I <sub>S5</sub>	-.6228	I <sub>O</sub>	-.5642
6	I <sub>S4</sub> /F	-.4400	Fixed w/o	-.6970	I <sub>S2</sub> /F	-.6624	I <sub>S3</sub> /F	-.6222	I <sub>FG</sub>	-.5642
7	I <sub>S2</sub> /F	-.4390	I <sub>S4</sub> /F	-.6836	I <sub>S3</sub> /F	-.6514	I <sub>N2</sub>	-.6194	Spurr w/	-.5603
8	Bitt. w/	-.4381	I <sub>S1</sub> /F	-.6271	Fixed w/	-.6405	Fixed w/	-.6111	Spurr w/o	-.5390
9	I <sub>S5</sub>	-.4336	I <sub>O</sub>	-.6694	Fixed w/o	-.6369	Fixed w/o	-.6090	Fixed w/o	-.5381
10	I <sub>S1</sub> /F	-.4313	I <sub>FG</sub>	-.6694	I <sub>B</sub>	-.6320	I <sub>S4</sub> /F	-.5900	Brown	+ .4571
11	I <sub>N</sub>	-.4262	I <sub>S3</sub> /F	-.6616	I <sub>FG</sub>	-.6290	I <sub>S1</sub> /F	-.5881	I <sub>S3</sub> /F	-.4406
12	I <sub>O</sub>	-.4254	I <sub>SJ2</sub>	-.6597	I <sub>O</sub>	-.6290	I <sub>B</sub>	-.5793	Bitt. w/o	-.4319
13	I <sub>FG</sub>	-.4253	I <sub>S5</sub>	-.6576	SBA	-.6278	I <sub>S1</sub>	-.5761	Bitt. w/	-.4155
14	Spurr w/	-.4210	I <sub>B</sub>	-.6518	I <sub>S1</sub>	-.6158	I <sub>S2</sub> /F	-.5692	I <sub>SJ3</sub>	+ .4131
15	I <sub>B</sub>	-.4178	Fixed w/	-.6441	I <sub>S5</sub>	-.6121	SBA	-.5479	I <sub>S1</sub>	+ .3853
16	Spurr w/o	-.4160	I <sub>SJ4</sub>	-.6441	I <sub>S1</sub> /F	-.6067	Spurr w/o	-.5475	I <sub>S1</sub> /F	-.3676
17	Fixed w/o	-.4127	Brown	+ .6430	I <sub>SJ3</sub>	-.5450	Brown mod.	-.5227	I <sub>S5</sub>	-.3366
18	Fixed w/	-.4123	I <sub>S1</sub>	-.6271	I <sub>SJ4</sub>	-.5250	Brown	+ .5141	SBA	-.3296
19	Brown mod.	+ .4055	Bitt. w/	-.6183	I <sub>SJ1</sub>	-.5063	I <sub>N</sub>	-.4789	I <sub>SJ4</sub>	+ .2854
20	I <sub>SJ3</sub>	-.3947	Bitt. w/o	-.6104	I <sub>SJ2</sub>	-.4849	Spurr w/	-.4726	Fixed w/	-.2802
21	I <sub>SJ1</sub>	-.3778	I <sub>N2</sub>	-.5916	Bitt. w/	-.4535	I <sub>SJ1</sub>	-.3762	I <sub>S4</sub> /F	-.1853
22	Brown	+ .3629	Brown mod.	+ .5890	Bitt. w/o	-.4497	I <sub>SJ3</sub>	-.3454	I <sub>S2</sub> /F	-.1321
23	I <sub>SJ4</sub>	-.2822	I <sub>N</sub>	-.5690	Brown	+ .3329	I <sub>SJ2</sub>	-.3092	I <sub>SJ1</sub>	+ .1201
24	I <sub>SJ2</sub>	-.2735	SBA	-.5493	Brown mod.	+ .2441	I <sub>SJ4</sub>	-.2749	I <sub>SJ2</sub>	-.0948

they differ only in the choice of a constant multiplier, the basal area factor (BAF).

If one should compute the 95 per cent confidence interval for any one of the correlation coefficients that are close to the average value, that interval would include virtually all of the other coefficients. For example, the correlation coefficient for the Steneker-Jarvis  $I_{8J2}$  for basal area growth of codominant longleaf pines, was  $-0.6597$ , Appendix B, Table 4. Twenty-eight trees were in that sample. The 0.95 confidence limits were  $-0.3800$  and  $0.8287$ . As can be seen in Table 2, this interval includes the correlation coefficients for every point density expression tested with that same set of data. This suggests that, with the sample sizes used here, few significant differences between correlation coefficients would be shown if suitable multiple range tests were available. Since differences between the various point density expressions are relatively small, more intensive sampling would be needed to confirm those differences if they really exist. It is doubtful that making such intensive tests would be of much practical value because there is no clear evidence that any specific expression would prove more useful than any other in a similar situation.

### Shape of Relationship

More often than not, the square root of the point density expression yielded a stronger correlation than either the unweighted point density expression or its square. However, as can be seen in Appendix B, Tables 2-24, in most instances the differences between the three correlation coefficients associated with different ways of expressing point density were small. It is doubtful that any of the differences were real. There may be a tendency toward curvilinearity of the relationship, but it is not strong. Linearity could probably be assumed in most cases without much loss of information.

### CONCLUSIONS

1. It appears that in general point density is not closely correlated with individual tree growth in either d.b.h. or basal area.
2. It appears that no point density expression is clearly better than the others as a predictor of tree growth. However, those based on overlapping C.I. Zones appear to perform generally better than the others while the Steneker-Jarvis and Brown expres-

sions appear to be less effective. The range in performance, however, is not great.

3. It appears that crown class must be recognized whenever point density is to be a factor in a study.

4. It appears that, at least in the case of the pine species studied here, edge effect penetrates deep into the stand. This can cause confounding in statistical studies involving the growth of individual trees.

5. It appears that the relationship between growth and point density may be curvilinear. However, this tendency is so slight that it probably can be ignored.

6. It appears that average stand density, measured in terms of basal area per acre, is just about as good a predictor of individual tree growth as is point density, especially in the case of the dominant and codominant crown classes. This indicates that the existing point density expressions are not functioning as expected and that, perhaps, a new approach to the problem of evaluation of competitive pressure on individual trees will have to be developed.

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### APPENDIX A

Computations for examples in Figures 1-10.

A.1. For Figure 1, basal area per acre, from fixed radius plots:

Plot area =  $\pi r^2 = 3.1416(15)^2 = 706.86$  sq. ft. or 0.016227 acre.

Trees inside plot:	D.b.h.	Basal area	Remarks
	In.	Sq. ft.	
1	10	0.545	
2	9	0.442	
3	5	0.136	
4	6	0.098	Half in and half out.
		1.221	Sq. ft., excluding sample tree.
S	7	0.267	Sample tree.
		1.488	Sq. ft., including sample tree.

Blow-up factor:  $1/0.016227 = 61.625686$

Basal area/acre, excluding sample tree = 61.625686

(1.221) = 75.2 sq. ft.

Basal area/acre, including sample tree = 61.625686

(1.488) = 91.7 sq. ft.

A.2. For Figure 1, the Steneker-Jarvis competition indices:

Trees inside plot:	$G_i$ Ft.	$G_i^2$	
1	13.75	203.1	
2	6.0	36.0	
3	8.0	64.0	
4	15.0	225.0	Half in and half out.

$$D_s = 7.0 \text{ inches; } D_s^2 = 49.0$$

$$I_{SJ1} = \sum_{i=1}^n (D_s/G_i) = (7/13.75) + (7/6.0) + (7/8.0) + (7/15.0) \\ = 3.017$$

$$I_{SJ2} = \sum_{i=1}^n (D_s/G_i^2) = (7/203.1) + (7/36.0) + (7/64.0) + (7/225.0) \\ = 0.369$$

$$I_{SJ3} = \sum_{i=1}^n (D_s^2/G_i) = (49.0/13.75) + (49.0/6.0) + (49.0/8.0) + \\ (49.0/15.0) = 21.122$$

$$I_{SJ4} = \sum_{i=1}^n (D_s^2/G_i^2) = (49.0/203.1) + (49.0/36.0) + (49.0/64.0) \\ + (49.0/225.0) = 2.586$$

A.3. For Figure 3, basal area per acre, using Spurr's angle-summation method:

Four trees will be used, the ones subtending the four largest angles (angle ranks 1 to 4).

Ranking the trees requires the computation of the sine of half of the subtended angle:

Tree	d.b.h.	Distance	$\text{Sine } \frac{\Theta}{2}$	$= (\frac{\text{d.b.h.}}{24}) / \text{Dist.}$	Rank
	In.	Ft.			
A.....	6.0	15	$(6/24)/15$	$= 0.01667$	6
B.....	10.0	13.75	$(10/24)/13.75$	$= 0.03030$	2
C.....	6.0	36	$(6/24)/36$	$= 0.00694$	9
D.....	8.0	28	$(8/24)/28$	$= 0.01191$	7
E.....	5.0	8	$(5/24)/8$	$= 0.02604$	3
F.....	8.0	19	$(8/24)/19$	$= 0.01754$	5
G.....	9.0	6	$(9/24)/6$	$= 0.06250$	1
H.....	9.0	20	$(9/24)/20$	$= 0.01875$	4
I.....	5.0	24	$(5/24)/24$	$= 0.00868$	8

Excluding the sample tree:

$$B_G = \frac{0.5 C r^2}{R^2} = \frac{0.5(43560)(9.0/24)^2}{6^2} = 85.06 \text{ sq. ft./acre}$$

$$B_B = \frac{1.5 C r^2}{R^2} = \frac{1.5(43560)(10.0/24)^2}{13.75^2} = 60.00$$

$$B_E = \frac{2.5 C r^2}{R^2} = \frac{2.5(43560)(5.0/24)^2}{8^2} = 73.85$$

$$B_H = \frac{3.5 C r^2}{R^2} = \frac{3.5(43560)(9.0/24)^2}{20^2} = \frac{53.59}{272.50}$$

$$\text{Basal area/acre} = 272.50/4 = 68.12 \text{ sq. ft.}$$

Including the sample tree:

$$B_G = \frac{1.5 C r^2}{R^2} = \frac{1.5(43560)(9.0/24)^2}{6^2} = 255.19 \text{ sq. ft./acre}$$

$$B_B = \frac{2.5 C r^2}{R^2} = \frac{2.5(43560)(10.0/24)^2}{13.75^2} = 99.99$$

$$B_E = \frac{3.5 C r^2}{R^2} = \frac{3.5(43560)(5.0/24)^2}{8^2} = 103.39$$

$$B_H = \frac{4.5 C r^2}{R^2} = \frac{4.5(43560)(9.0/24)^2}{20^2} = \frac{68.90}{527.47}$$

$$\text{Basal area/acre} = 527.47/4 = 131.87 \text{ sq. ft.}$$

A.4. For Figure 6, the Staebler indices:

$$D_s = 7.0''$$

$$D_a = \sum_{i=1}^{10} D_i/10 = 73/10 = 7.3'' \text{ d.b.h. of average tree in stand.}$$

$$A = a D + k = 2 D + 2 = \text{diameter of C.I.Z. in feet.}$$

$$F = [(a/2)(D_s + D_a) + k]^2/10 = [(2/2)(7.0 + 7.3) + 2]^2/10 = 26.57 \text{ which rounds to } 27.$$

$$d_A = 5.2'$$

$$d_B = 12.0'$$

$$d_C = 6.0'$$

$$I_{S1} = \sum_{i=1}^3 d_i = 5.2 + 6.0 + 12.0 = 23.2 \text{ feet}$$

$$I_{S1}/F = \sum_{i=1}^n d_i/F = 23.2/27 = 0.86$$

$$I_{S2}/F = \sum_{i=1}^n d_i^2/F = 207.04/27 = 7.67$$

$$I_{S3}/F = \sum_{i=1}^n (d_i D_c)/F = [5.2(10) + 6.0(5) + 12.0(9)]/27 = 7.15$$

$$I_{S4}/F = \frac{\sum_{i=1}^n (d_i D_c)/F}{65.05} = [(5.2)^2(10) + (6.0)^2(5) + (12.0)^2(9)]/27 =$$

$$I_{S5} = \sum_{i=1}^n d_i/A_s = 23.2/16 = 1.45 \text{ or } 145\%.$$

A.5. For Figure 7, the Newnham indices:

$$A = 2D + 2 = \text{diameter of C.I.Z. in feet}$$

$$A_S = 2(7.0) + 2 = 16$$

$$A_A = 2(10.0) + 2 = 22$$

$$A_B = 2(9.0) + 2 = 20$$

$$A_C = 2(5.0) + 2 = 12$$

$$\alpha_A = 105^\circ$$

$$\alpha_B = 177^\circ$$

$$\alpha_C = 88^\circ$$

$$I_N = \frac{1}{360^\circ} \sum_{i=1}^3 [\alpha_i (A_i/A_s)]$$

$$= \frac{1}{360} [105(22/16) + 177(20/16) + 88(12/16)] = \frac{431.625}{360}$$

$$= 1.20 \text{ or } 120\%$$

$$I_{N2} = \frac{1}{360^\circ} \sum_{i=1}^3 (\alpha_i) = \frac{1}{360} (105 + 177 + 88)$$

$$= \frac{370}{360} = 1.03 \text{ or } 103\%.$$

A.6. For Figure 10, the Fritts-Gerrard index:

The areas in the cross-hatched overlaps could be measured using a planimeter or a dot grid, or they could be computed using conventional mensurational formulae. The latter procedure was used in this study and is described below.

Angles from the sample tree, as were used in Newnham's method:

$$\alpha_A = 105^\circ \text{ or } 1.8326 \text{ radians}$$

$$\alpha_B = 177^\circ \text{ or } 3.0892$$

$$\alpha_C = 88^\circ \text{ or } 1.5359$$

Equivalent angles measured from the centers of the competing trees:

$$\beta_A = 72^\circ \text{ or } 1.2566 \text{ radians}$$

$$\beta_B = 119^\circ \text{ or } 2.0769$$

$$\beta_C = 135^\circ \text{ or } 2.3562$$



Radii of C. I. Zones:

$$A_S/2 = 16/2 = 8.0 \text{ feet}$$

$$A_A/2 = 22/2 = 11.0$$

$$A_B/2 = 20/2 = 10.0$$

$$A_C/2 = 12/2 = 6.0$$

Areas of segments of C. I. Zones on sides of overlap areas toward the sample tree:

$$\begin{aligned} U_{SA} &= \frac{\alpha_A(A_S/2)^2}{2} - (A_S/2)^2 \left[ \cos\left(\frac{\alpha_A}{2}\right) \sin\left(\frac{\alpha_A}{2}\right) \right] \\ &= \frac{1.8326(8.0)^2}{2} - (8.0)^2 \left[ \cos\left(\frac{105^\circ}{2}\right) \sin\left(\frac{105^\circ}{2}\right) \right] \\ &= 27.73 \text{ sq. ft.} \end{aligned}$$

$$\begin{aligned} U_{SB} &= \frac{3.0892(8.0)^2}{2} - (8.0)^2 \left[ \cos\left(\frac{177^\circ}{2}\right) \sin\left(\frac{177^\circ}{2}\right) \right] \\ &= 97.18 \text{ sq. ft.} \end{aligned}$$

$$\begin{aligned} U_{SC} &= \frac{1.5359(8.0)^2}{2} - (8.0)^2 \left[ \cos\left(\frac{88^\circ}{2}\right) \sin\left(\frac{88^\circ}{2}\right) \right] \\ &= 17.17 \text{ sq. ft.} \end{aligned}$$

Areas of segments of C. I. Zones on sides of overlap areas away from the sample tree:

$$\begin{aligned} U_A &= \frac{\beta_A(A_A/2)^2}{2} - (A_A/2)^2 \left[ \cos\left(\frac{\beta_A}{2}\right) \sin\left(\frac{\beta_A}{2}\right) \right] \\ &= \frac{1.2566(11.0)^2}{2} - (11.0)^2 \left[ \cos\left(\frac{72^\circ}{2}\right) \sin\left(\frac{72^\circ}{2}\right) \right] \\ &= 18.48 \text{ sq. ft.} \end{aligned}$$

$$\begin{aligned} U_B &= \frac{2.0769(10.0)^2}{2} - (10.0)^2 \left[ \cos\left(\frac{119^\circ}{2}\right) \sin\left(\frac{119^\circ}{2}\right) \right] \\ &= 60.12 \text{ sq. ft.} \end{aligned}$$

$$\begin{aligned} U_C &= \frac{2.3562(6.0)^2}{2} - (6.0)^2 \left[ \cos\left(\frac{135^\circ}{2}\right) \sin\left(\frac{135^\circ}{2}\right) \right] \\ &= 29.68 \text{ sq. ft.} \end{aligned}$$

Total: 250.36 sq. ft. overlap

Area of sample tree C.I.Z.

$$S = \pi (A_S/2)^2 = 3.1416 (8.0)^2 = 201.06 \text{ sq. ft.}$$

$$I_{FG} = \frac{100}{S} \sum_{i=1}^n O_i = \left( \frac{100}{201.06} \right) \Sigma 250.36 = 124.52\%$$

A.7. For Figure 10, the Bella index:

K was arbitrarily set equal to 1. Therefore, the C. I. Zones were equal in size to those used for the previous calculations.

X was arbitrarily set equal to 2.

The overlapped areas in sq. ft.:

$$\begin{array}{rcl} \text{Between A and the sample tree} & = & 27.73 + 18.48 = 46.21 \\ \text{B} & = & 97.18 + 60.12 = 157.30 \\ \text{C} & = & 17.17 + 29.68 = 46.85 \end{array}$$

The  $O_i(D_i/D_s)^x$  values:

$$\begin{array}{rcl} 46.21 (10.0/7.0)^2 & = & 94.31 \\ 157.30 (9.0/7.0)^2 & = & 260.03 \\ 46.85 (5.0/7.0)^2 & = & 23.90 \\ \hline & & 378.24 \end{array}$$

$$I_B = \frac{1}{S} \sum_{i=1}^n [O_i(D_i/D_s)^x] = 378.24/201.06 = 1.88 \text{ or } 188\%.$$

A.8. For Figure 10, the Opie index:

Opie's "m" is equivalent to "k" above. Opie did not use an additive term when defining his C. I. Zones but the +1 used here should not invalidate the procedure. The BAF, using Equation (30).

$$\text{BAF} = 43560/(25k)^2 = 43560/[(25)(1)]^2 = 69.70$$

$$I_o = \frac{\text{BAF}}{S} \sum_{i=1}^n O_i = \frac{69.70}{201.06} (250.36)$$

$$= 86.79 \text{ sq. ft./acre of basal area.}$$

APPENDIX B

Figures

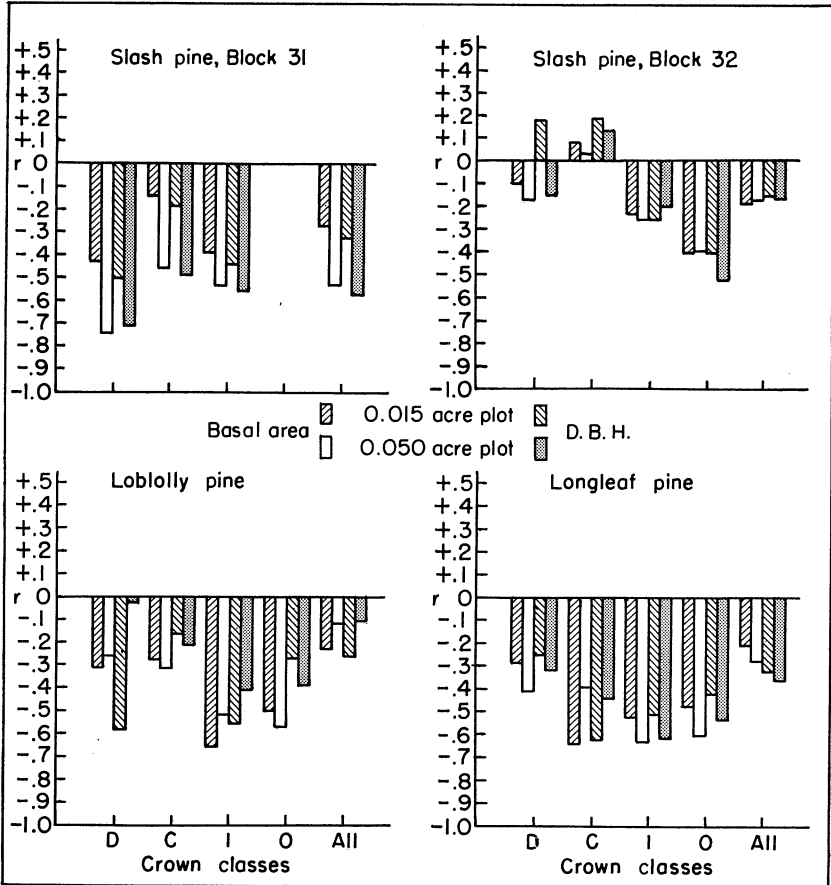


FIG. 14. Correlation coefficients associated with basal areas per acre from fixed radius plots, with sample tree.

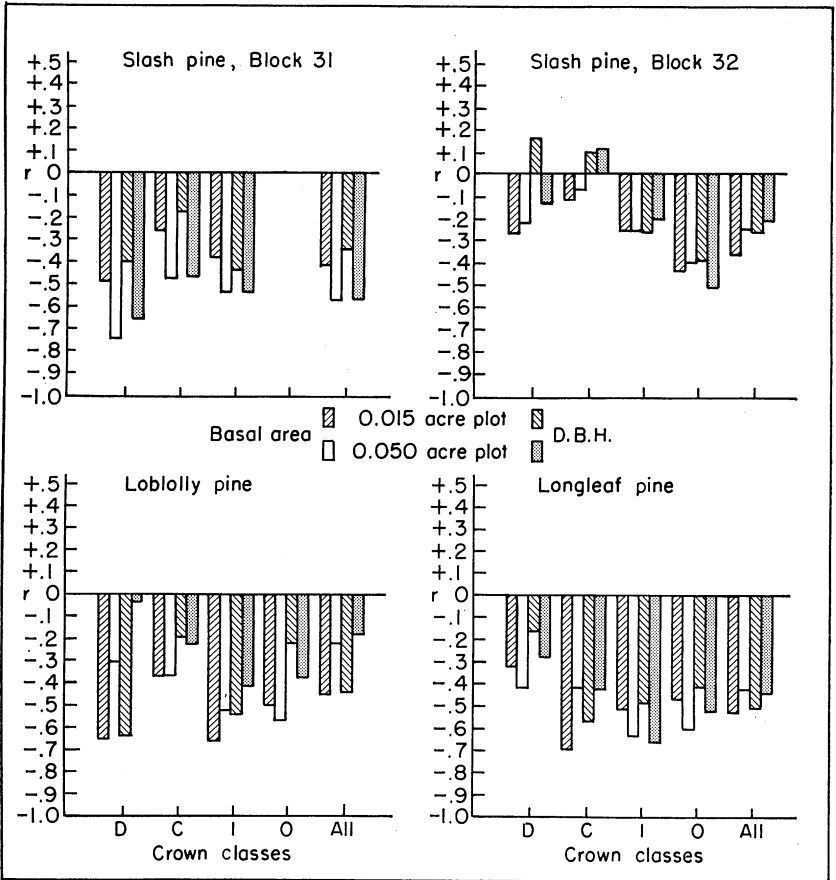


FIG. 15. Correlation coefficients associated with basal areas per acre from fixed radius plots, without sample tree.

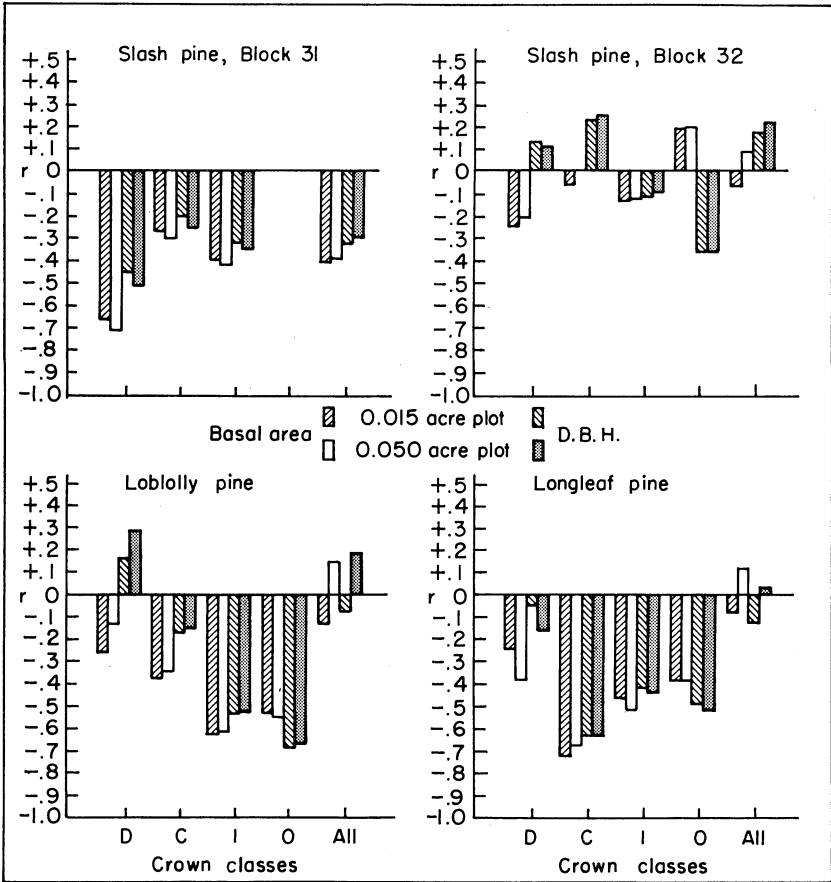


FIG. 16. Correlation coefficients associated with the Steneker and Jarvis competition index  $I_{SJ1}$ .

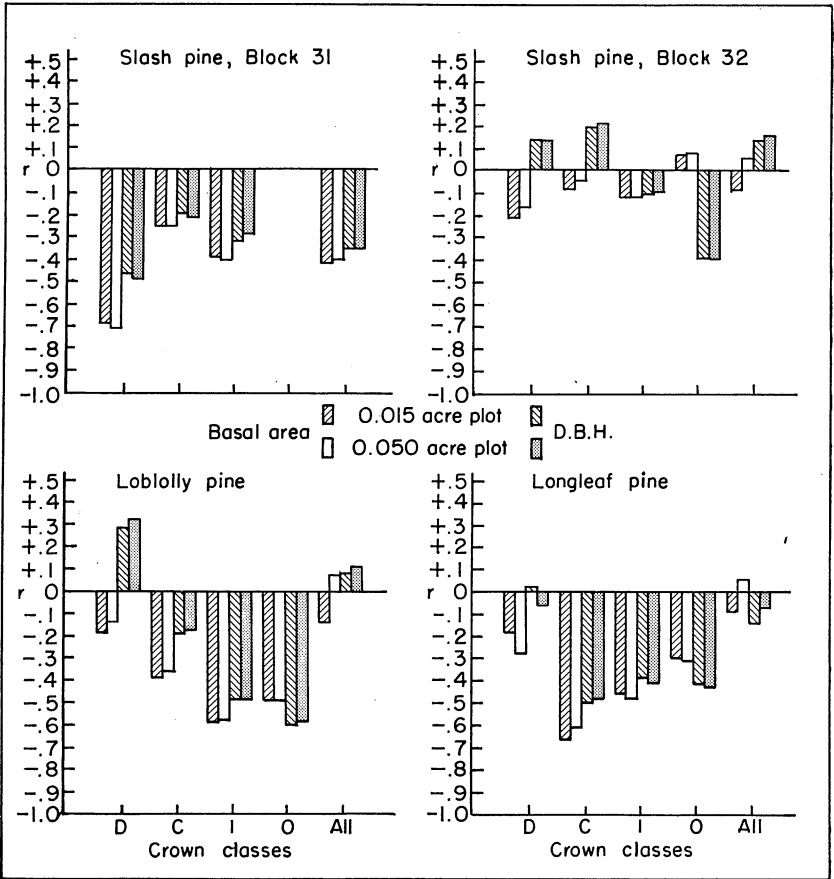


FIG. 17. Correlation coefficients associated with the Steneker and Jarvis competition index  $I_{SJ2}$ .

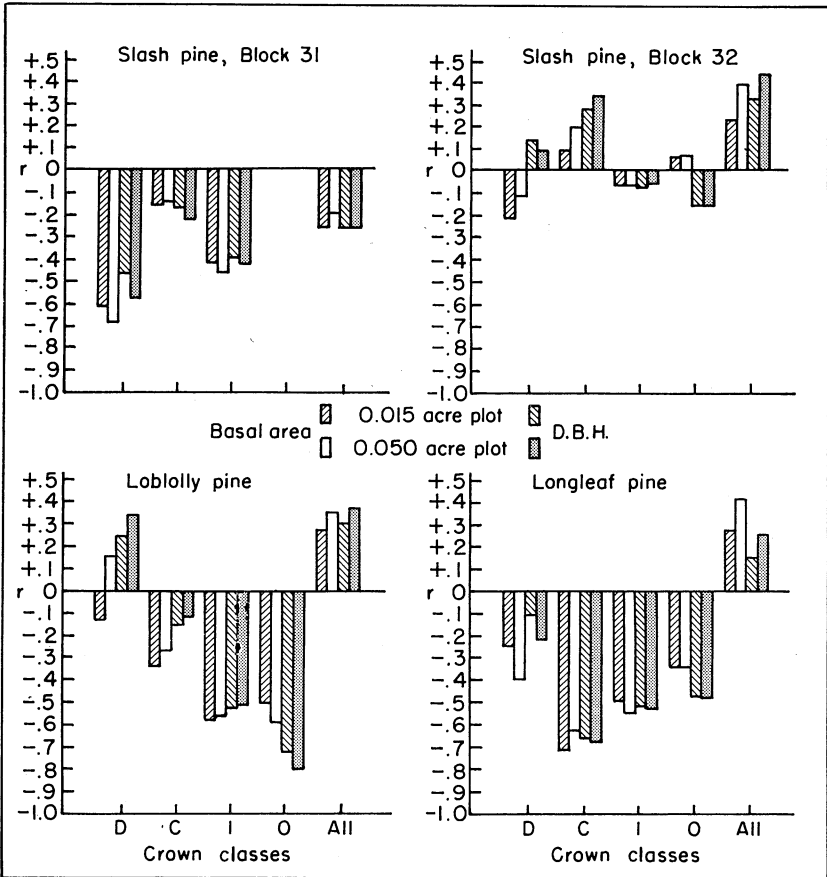


FIG. 18. Correlation coefficients associated with the Steneker and Jarvis competition index  $I_{SJ3}$ .

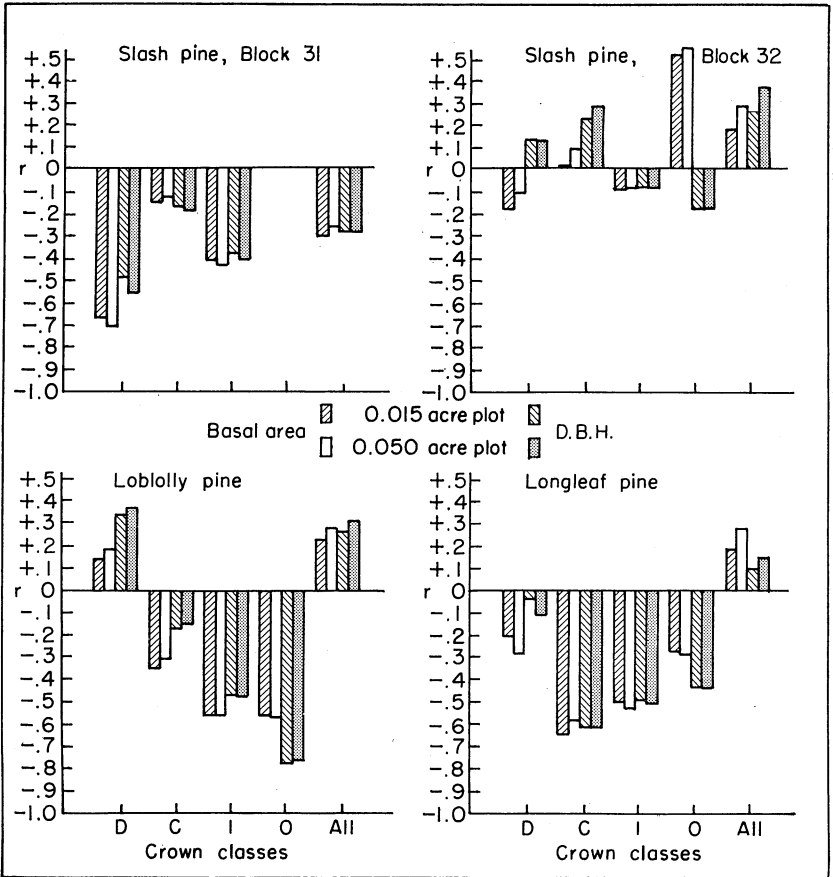


FIG. 19. Correlation coefficients associated with the Steneker and Jarvis competition index  $I_{SJ4}$ .



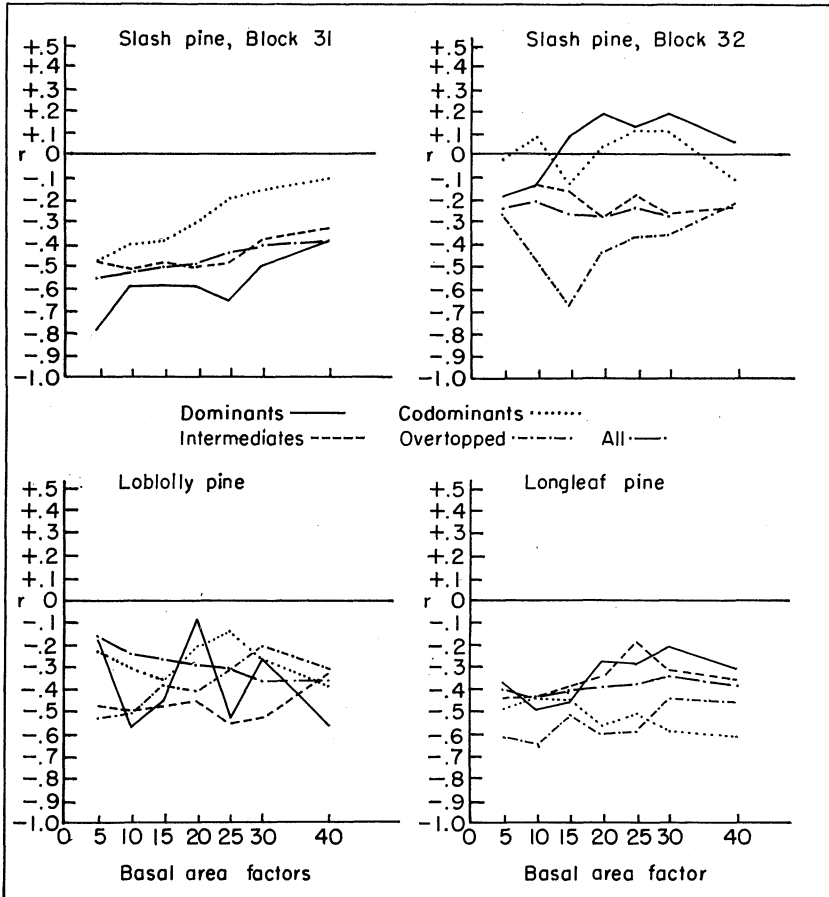


FIG. 20. Correlation coefficients associated with basal area growth and the Bit-terlich method, with sample tree.

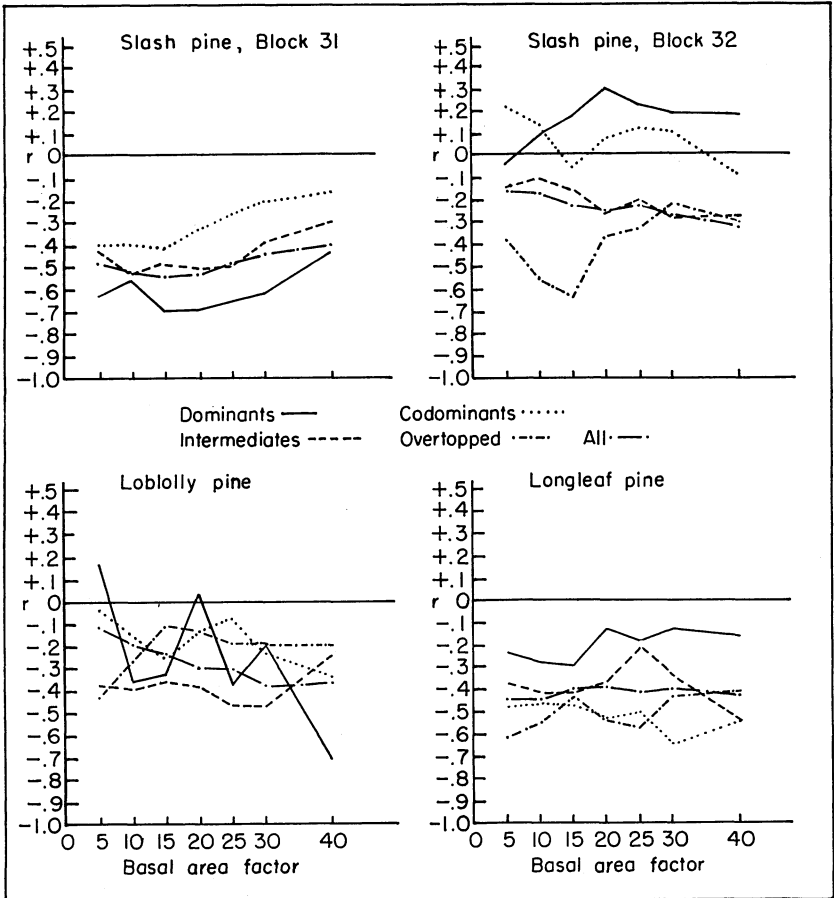


FIG. 21. Correlation coefficients associated with d.b.h. growth and the Bitterlich method, with sample tree.

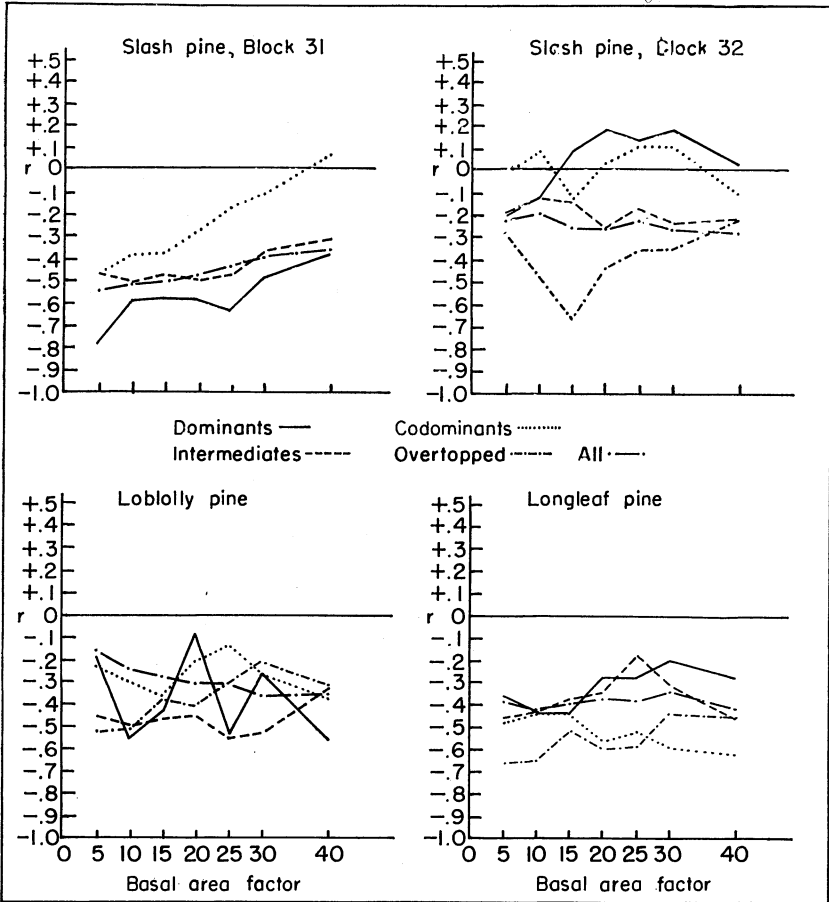


FIG. 22. Correlation coefficients associated with basal area growth and the Bitterlich method, without sample tree.

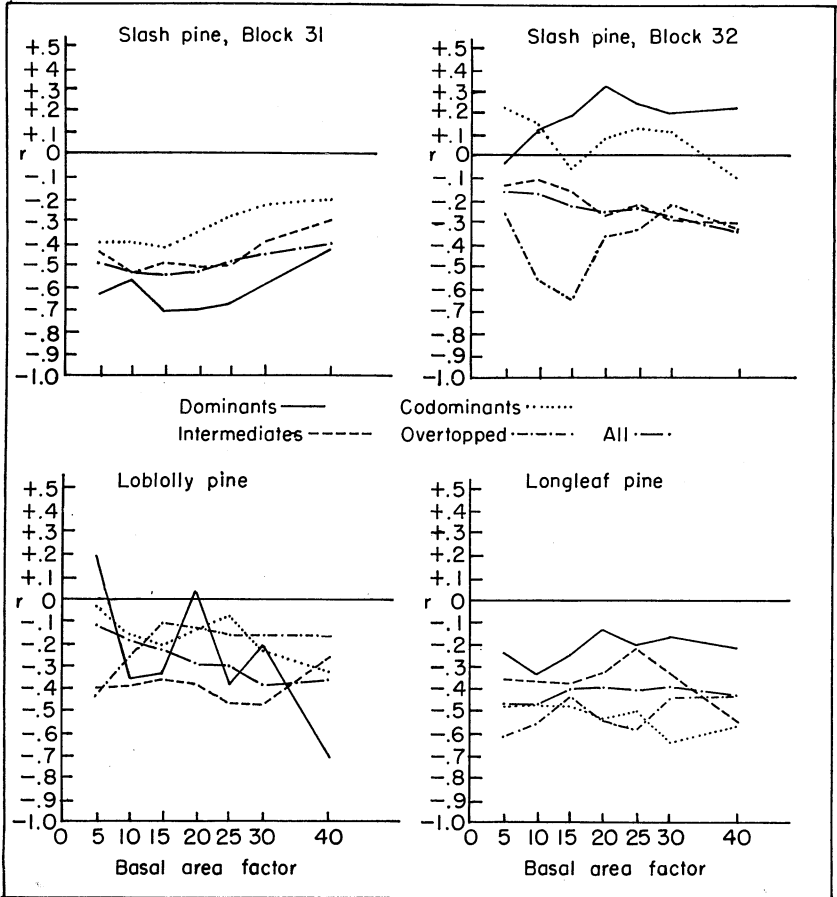


FIG. 23. Correlation coefficients associated with d.b.h. growth and the Bitterlich method, without sample tree.

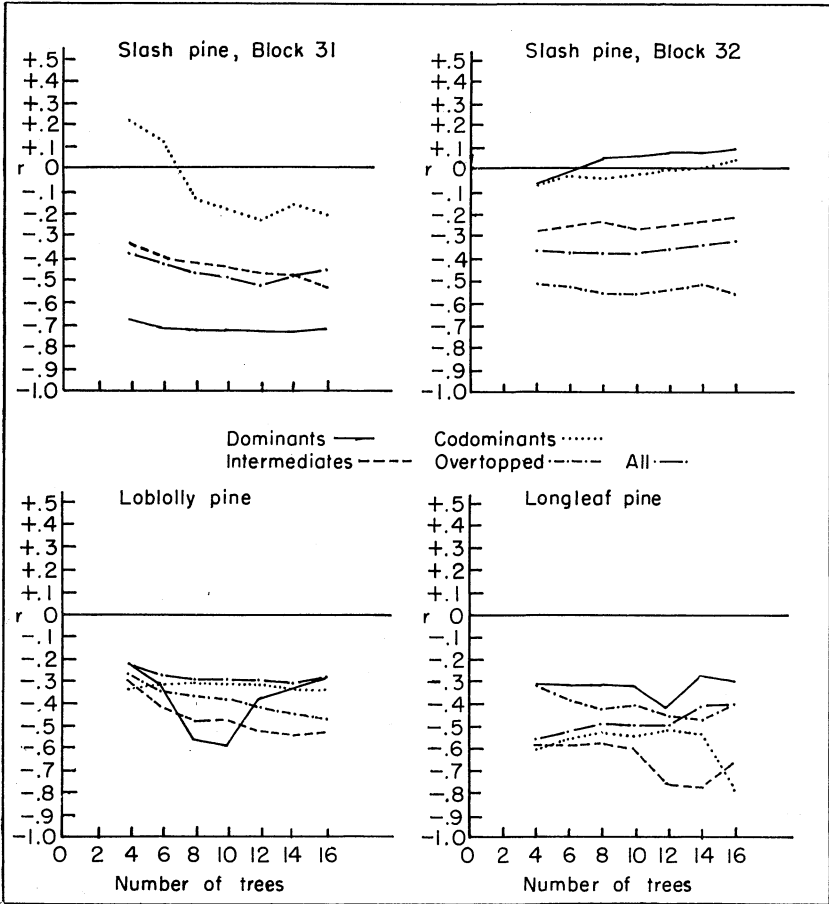


FIG. 24. Correlation coefficients associated with basal area growth and the Spurr method, with sample tree.

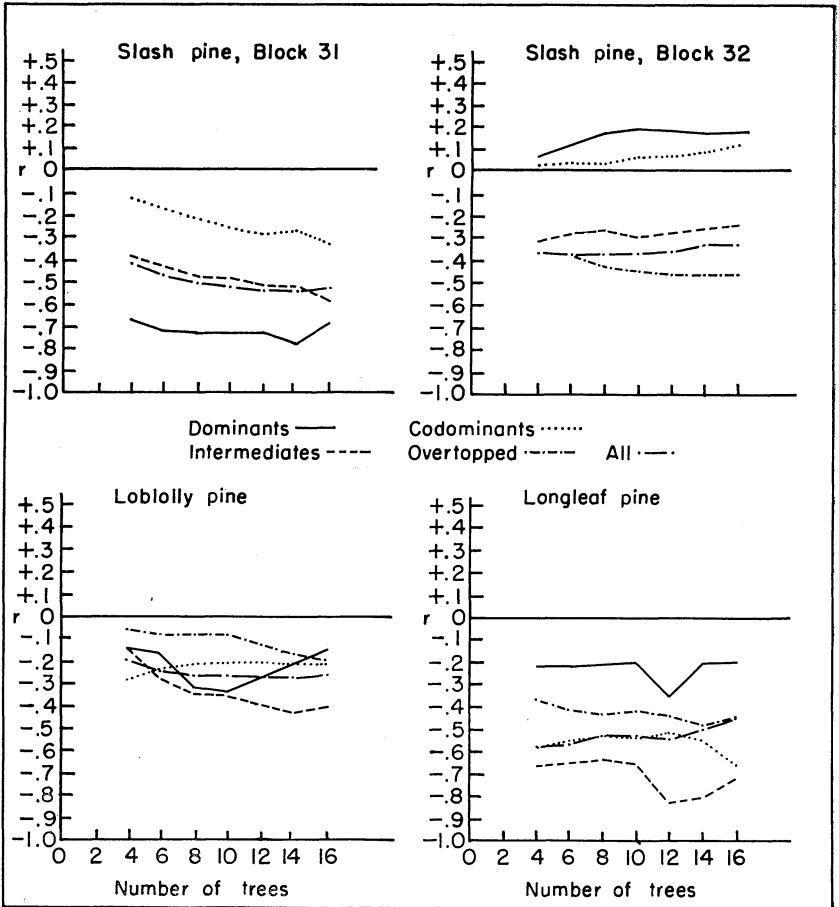


FIG. 25. Correlation coefficients associated with d.b.h. growth and the Spurr method, with sample tree.

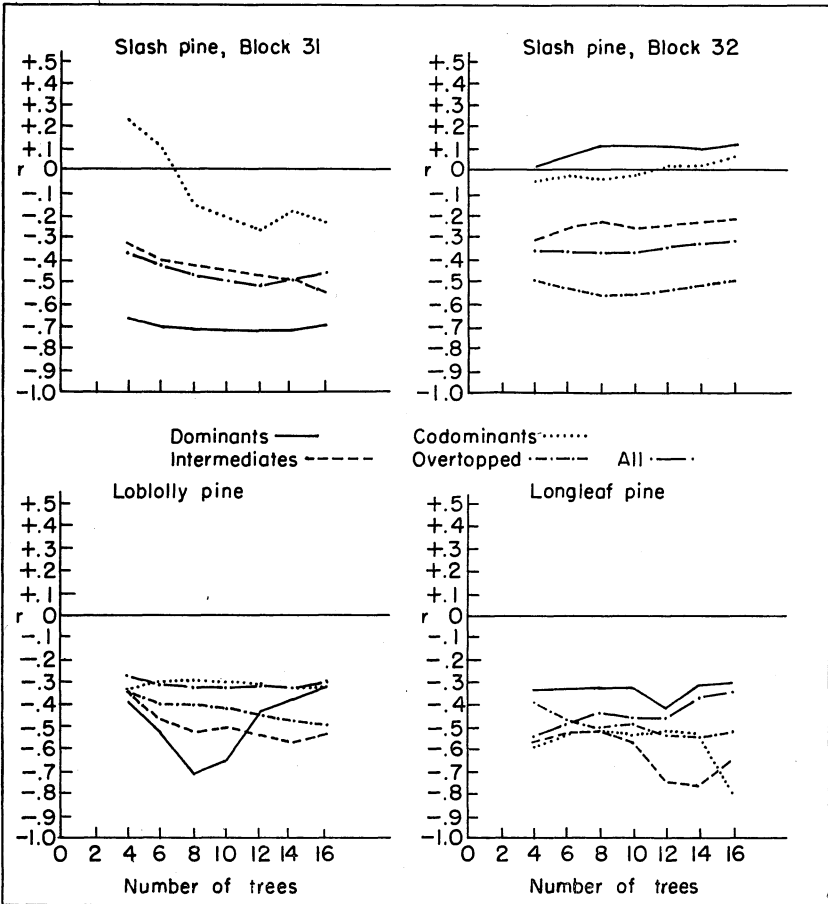


FIG. 26. Correlation coefficients associated with basal area growth and the Spurr method, without sample tree.

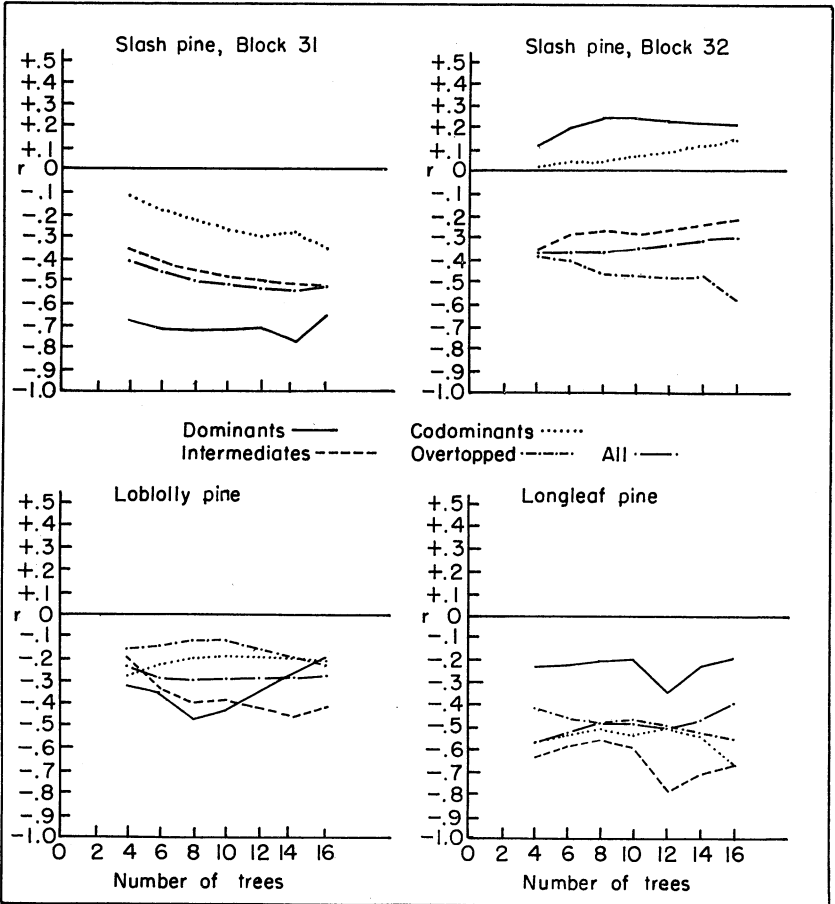


FIG. 27. Correlation coefficients associated with d.b.h. growth and the Spurr method, without sample tree.



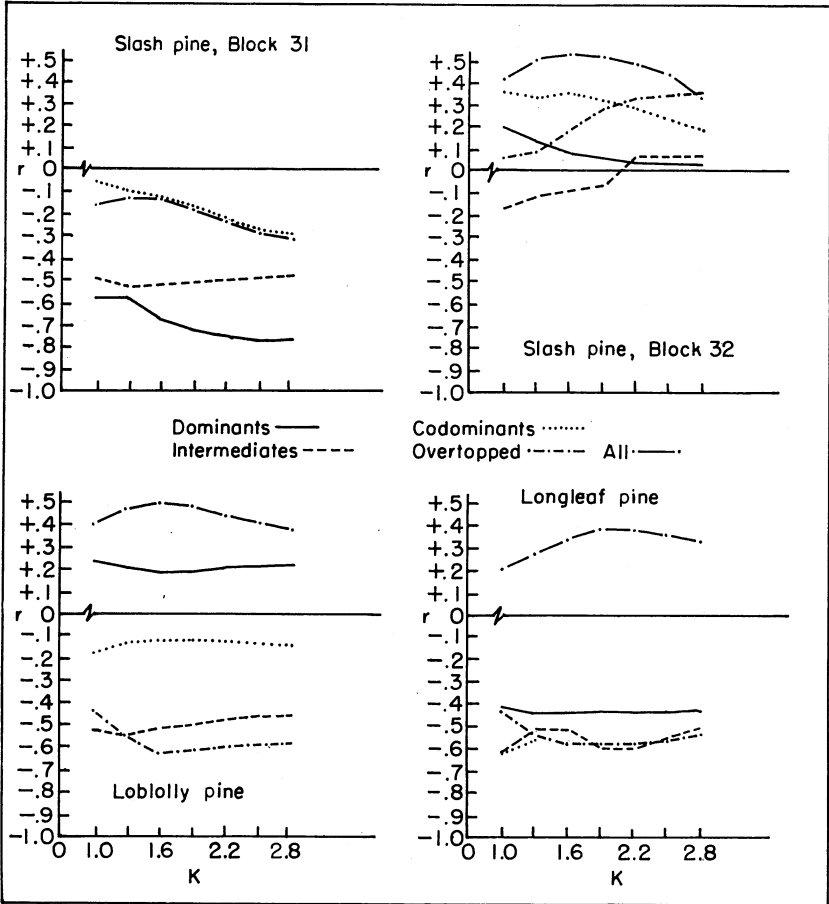


FIG. 28. Correlation coefficients associated with basal area growth and the Staebler competition index  $I_{S1}$ .

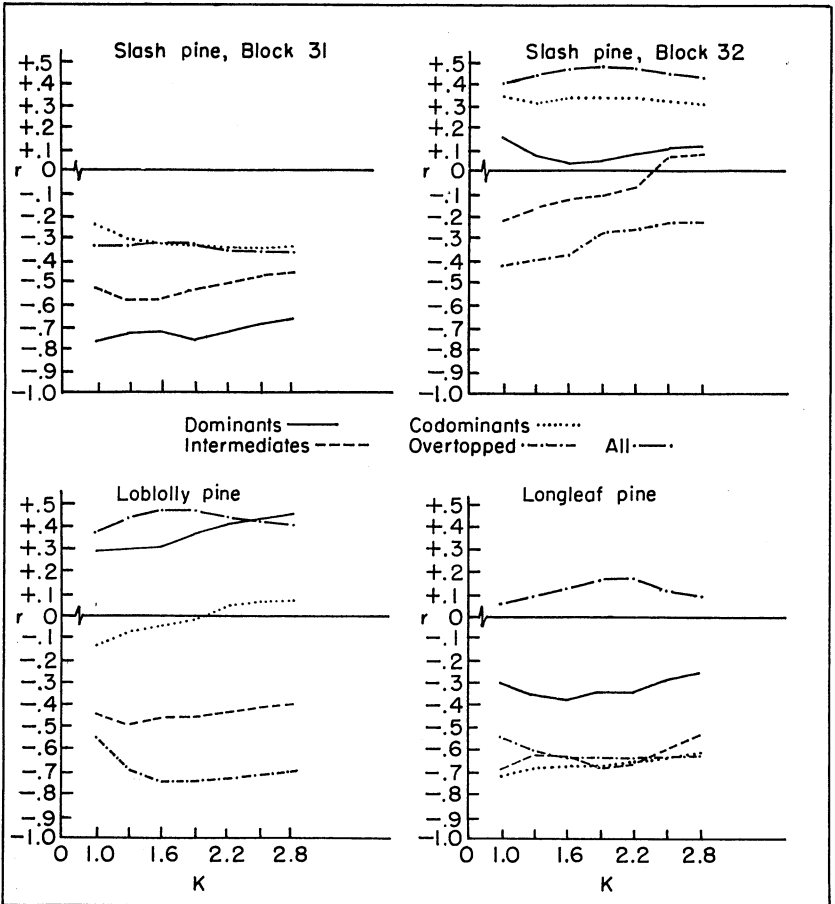


FIG. 29. Correlation coefficients associated with d.b.h. growth and the Staebler competition index  $I_{S1}$ .

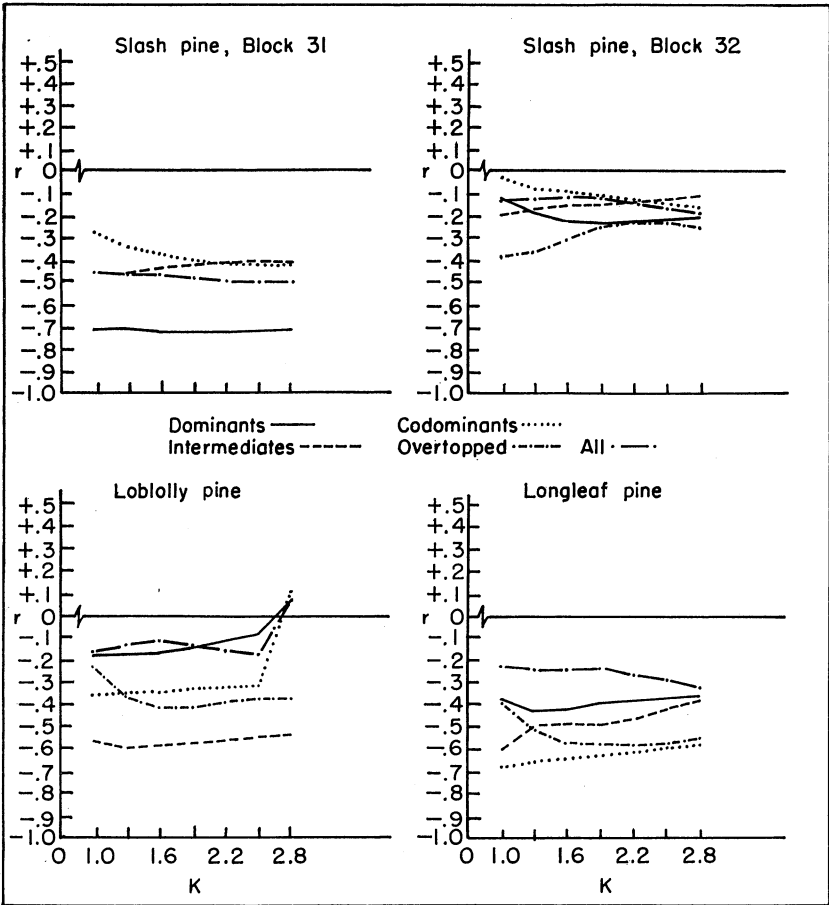


FIG. 30. Correlation coefficients associated with basal area growth and the Staebler competition index  $I_{S1}/F$ .

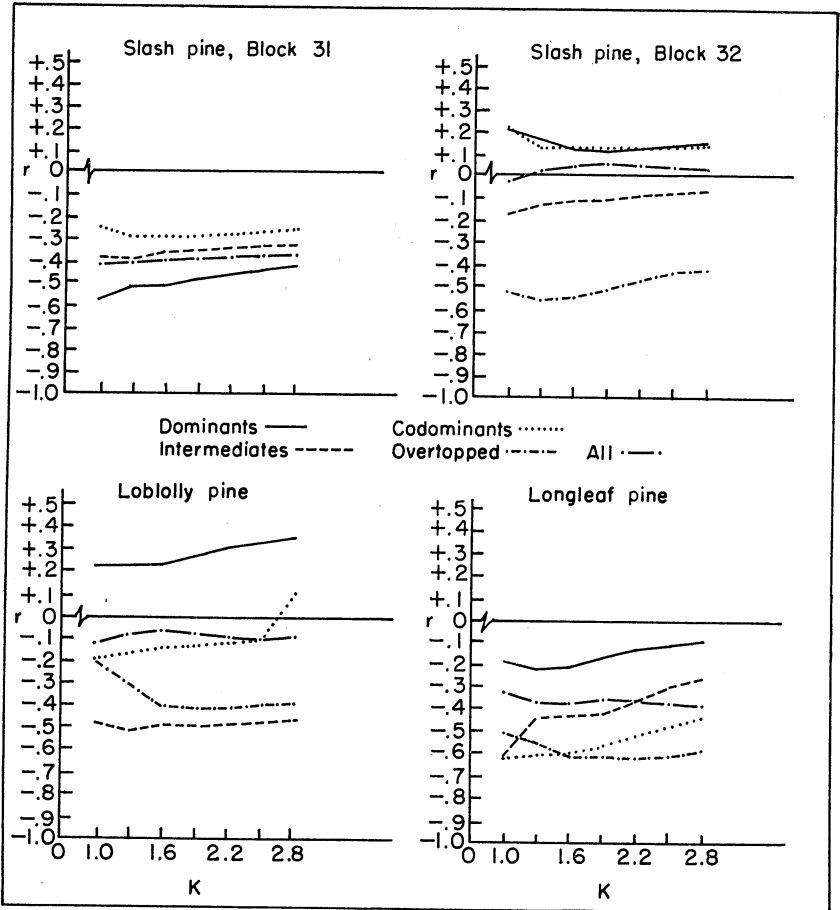


FIG. 31. Correlation coefficients associated with d.b.h. growth and the Staebler competition index  $S_1/F$ .

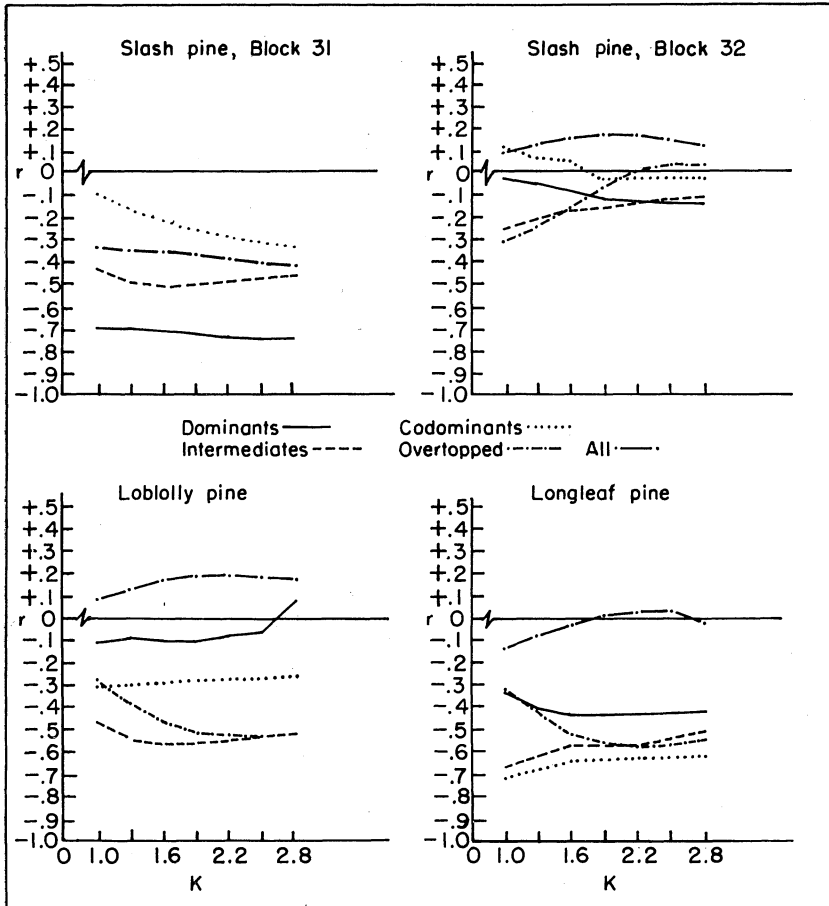


FIG. 32. Correlation coefficients associated with basal area growth and the Staebler competition index  $I_{S_2}/F$ .

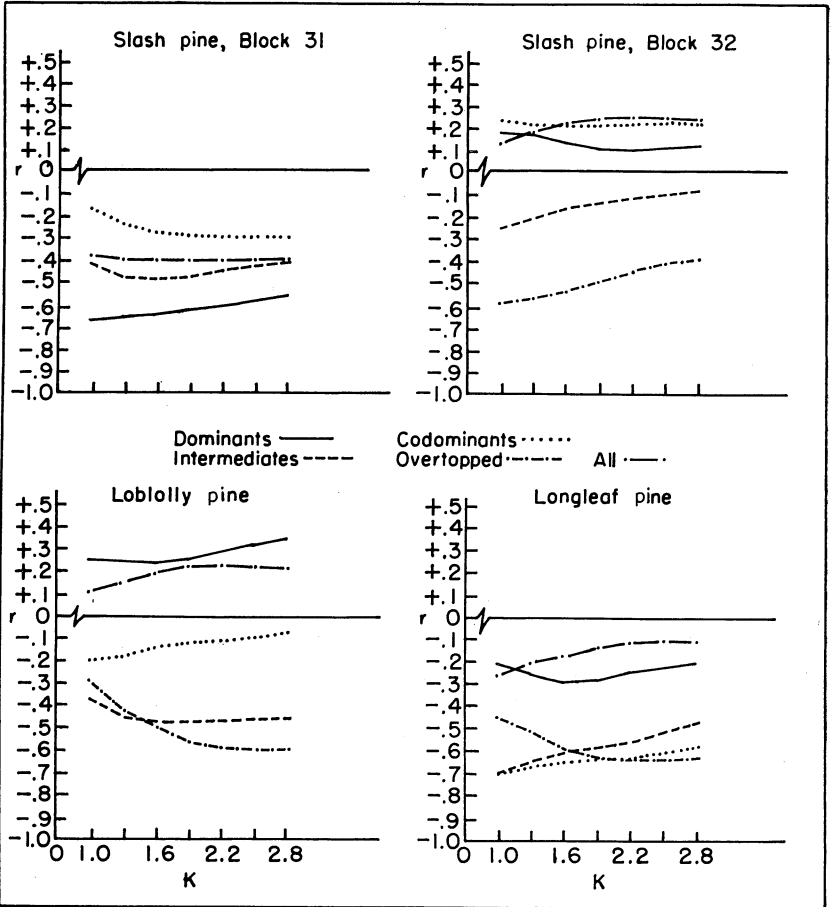


FIG. 33. Correlation coefficients associated with d.b.h. growth and the Staebler competition index  $I_{S_2}/F$ .

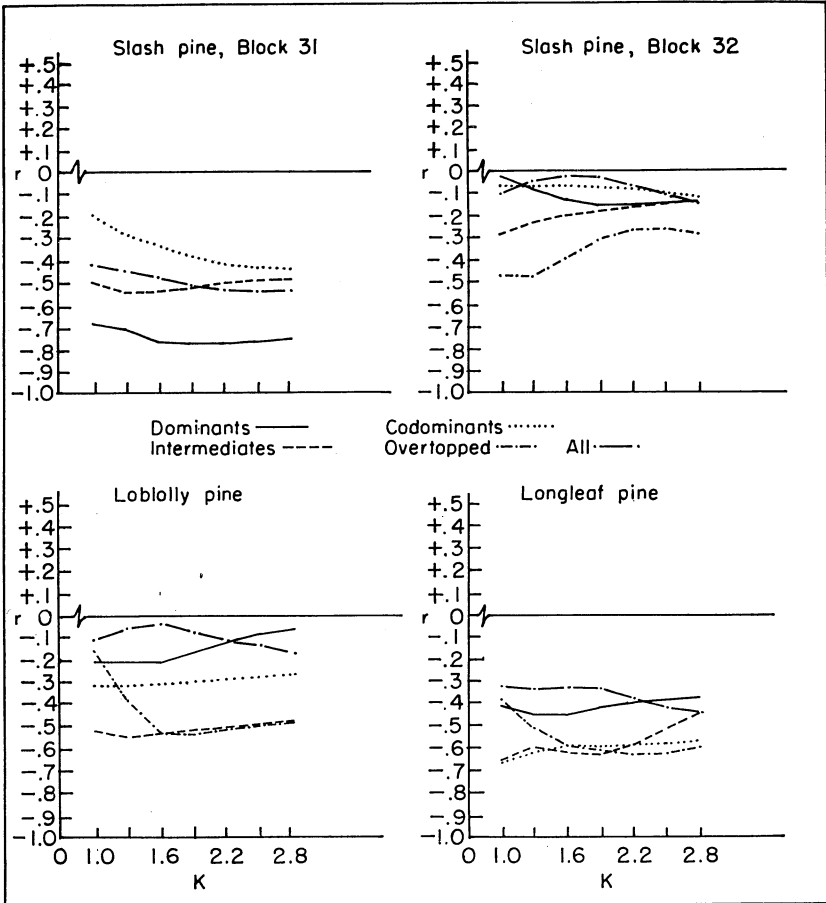


FIG. 34. Correlation coefficients associated with basal area growth and the Staebler competition index  $I_{S_3}/F$ .

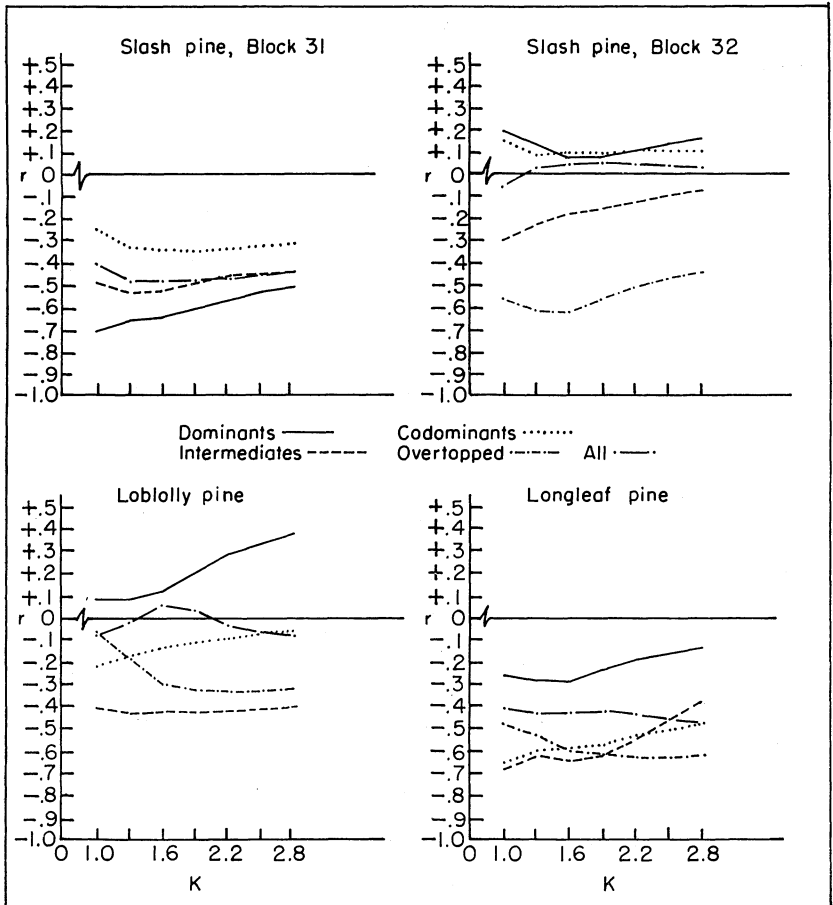


FIG. 35. Correlation coefficients associated with d.b.h. growth and the Staebler competition index  $I_{S3}/F$ .



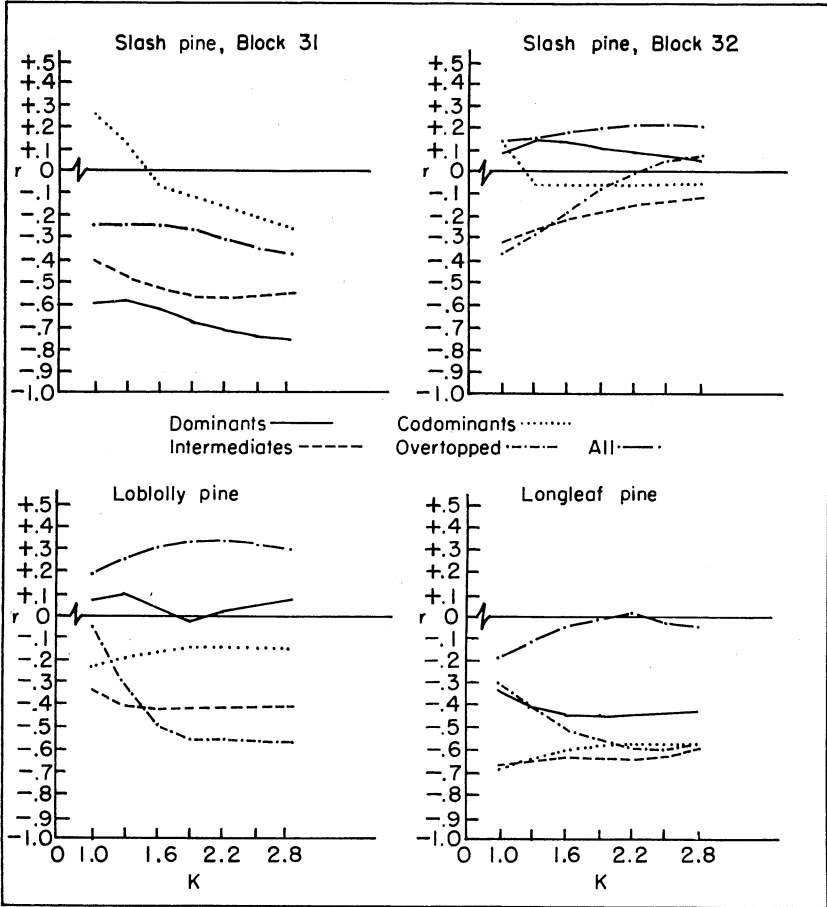


FIG. 36. Correlation coefficients associated with basal area growth and the Staebler competition index  $I_{S_4}/F$ .

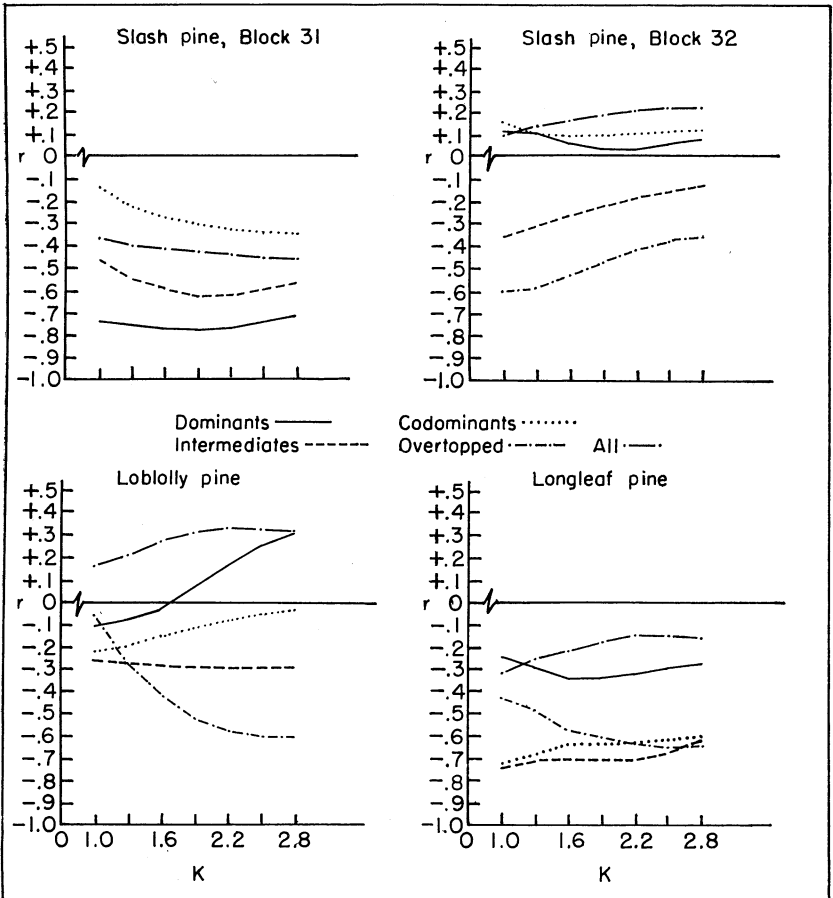


FIG. 37. Correlation coefficients associated with d.b.h. growth and the Staebler competition index  $I_{S_4}/F$ .

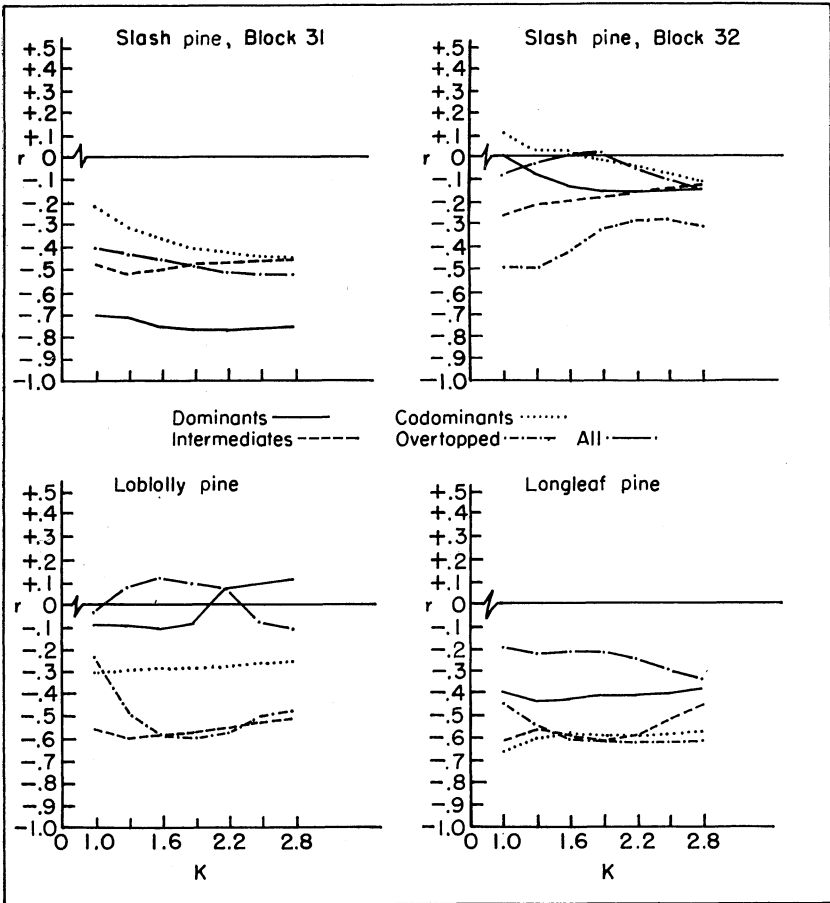


FIG. 38. Correlation coefficients associated with basal area growth and the Staebler competition index  $I_{S5}$ .

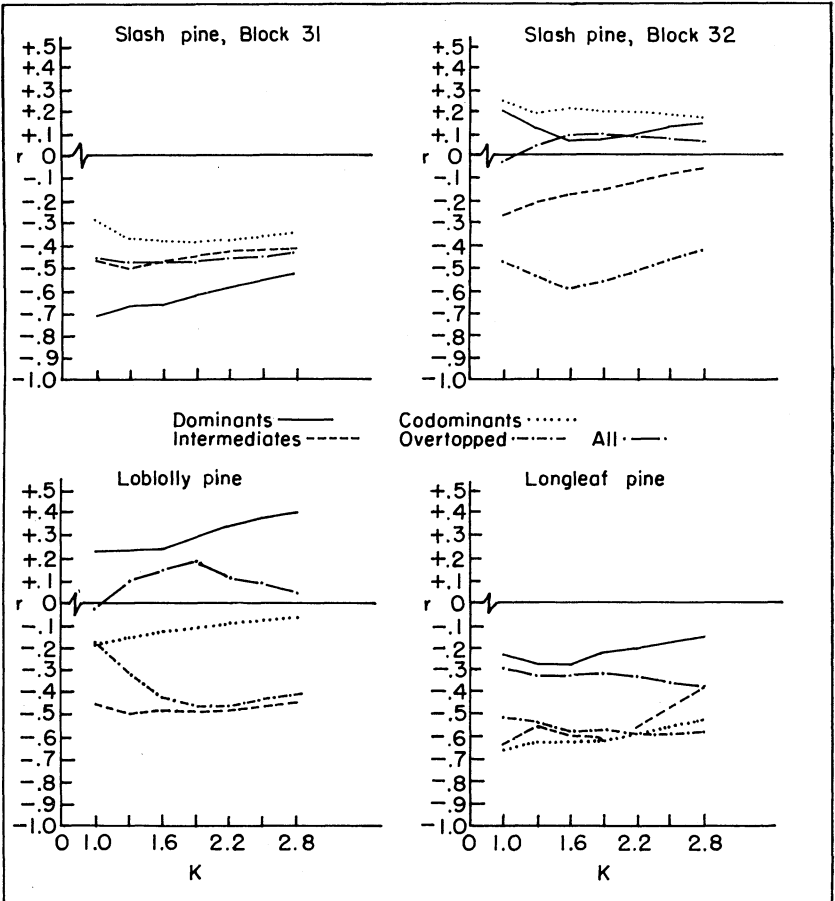


FIG. 39. Correlation coefficients associated with d.b.h. growth and the Staebler competition index  $I_{S5}$ .

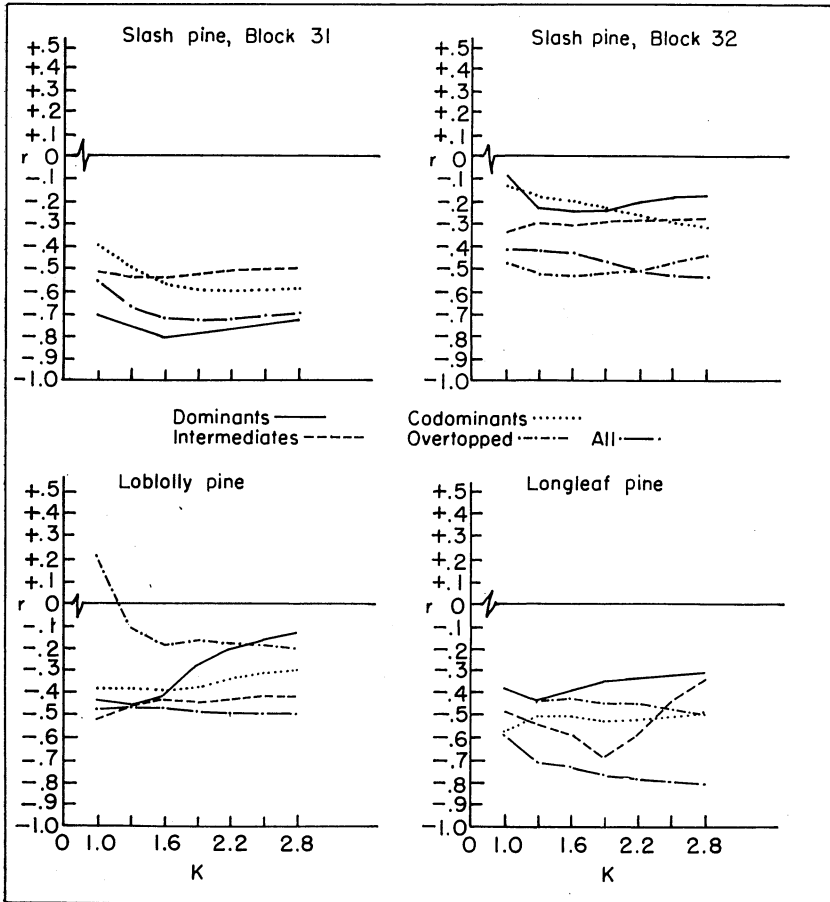


FIG. 40. Correlation coefficients associated with basal area growth and the Newnham competition index  $I_N$ .

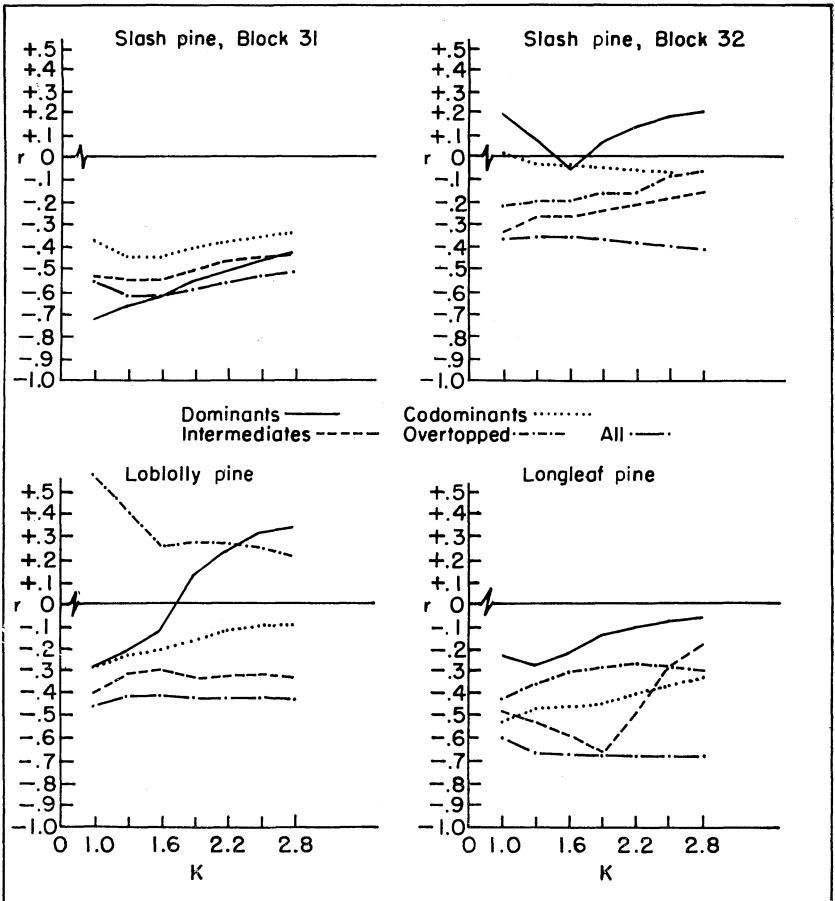


FIG. 41. Correlation coefficients associated with d.b.h. growth and the Newnham competition index  $I_N$ .

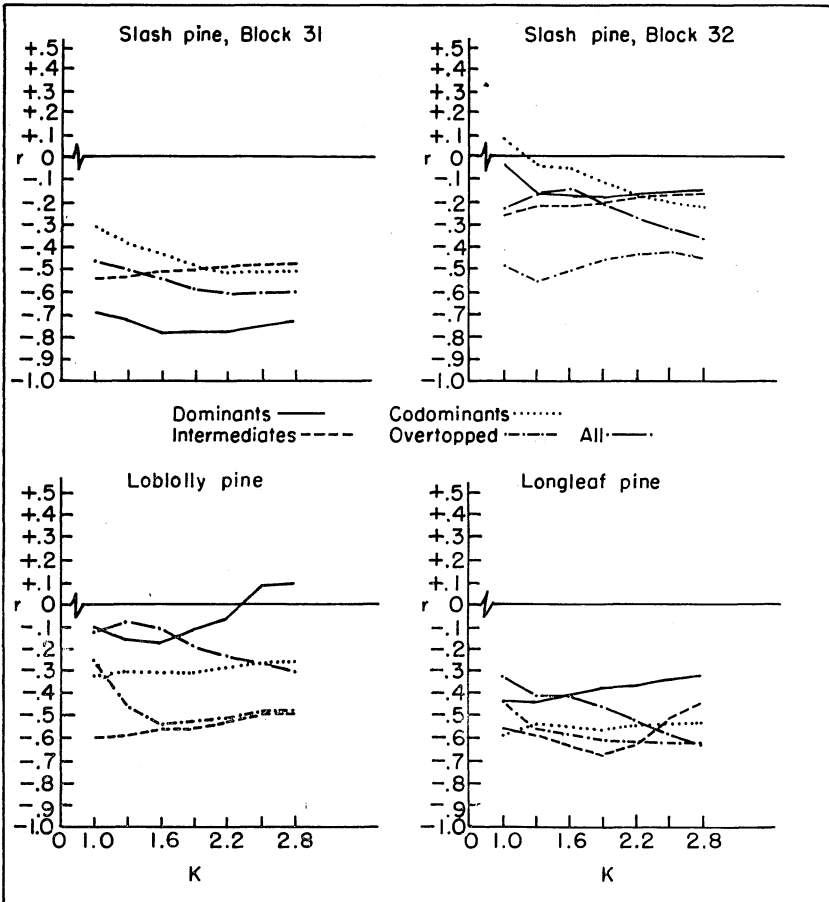


FIG. 42. Correlation coefficients associated with basal area growth and the N<sub>2</sub> index.

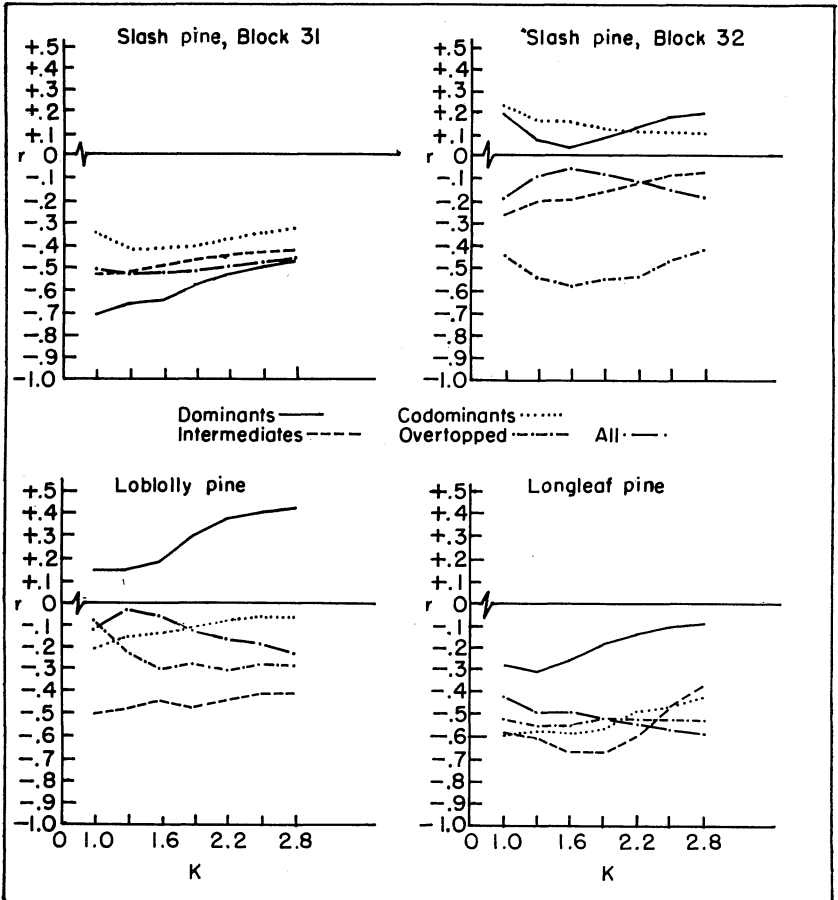


FIG. 43. Correlation coefficients associated with d.b.h. growth and the Newnham competition index  $I_{N2}$ .



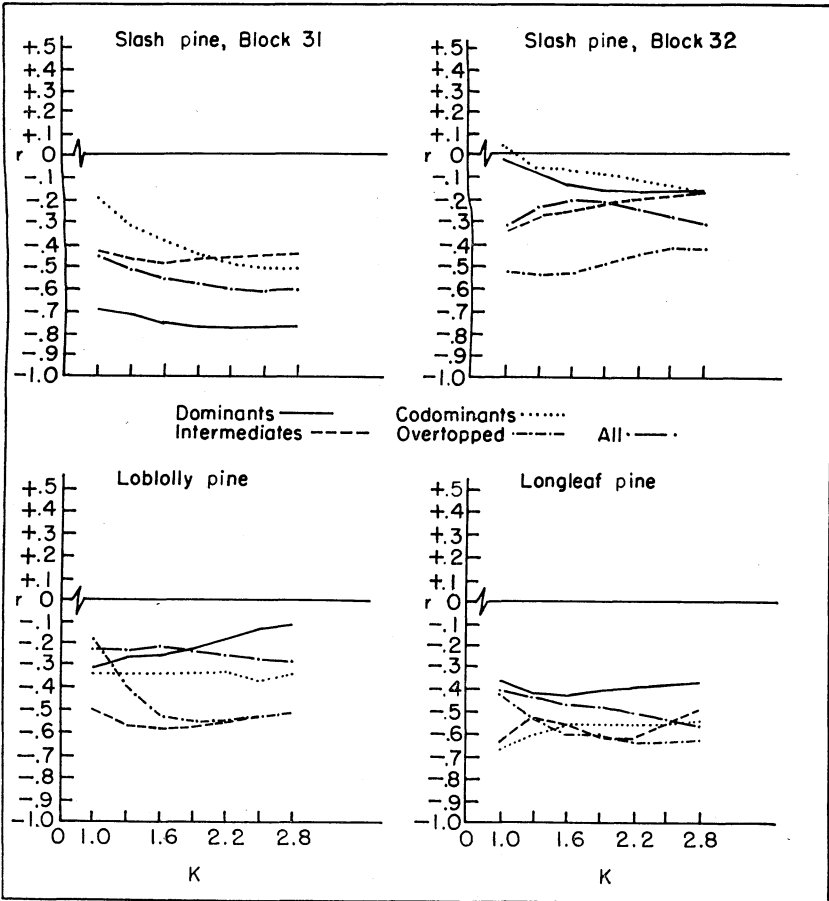


FIG. 44. Correlation coefficients associated with basal area growth and the Fritts-Gerrard competition index  $I_{FG}$ .

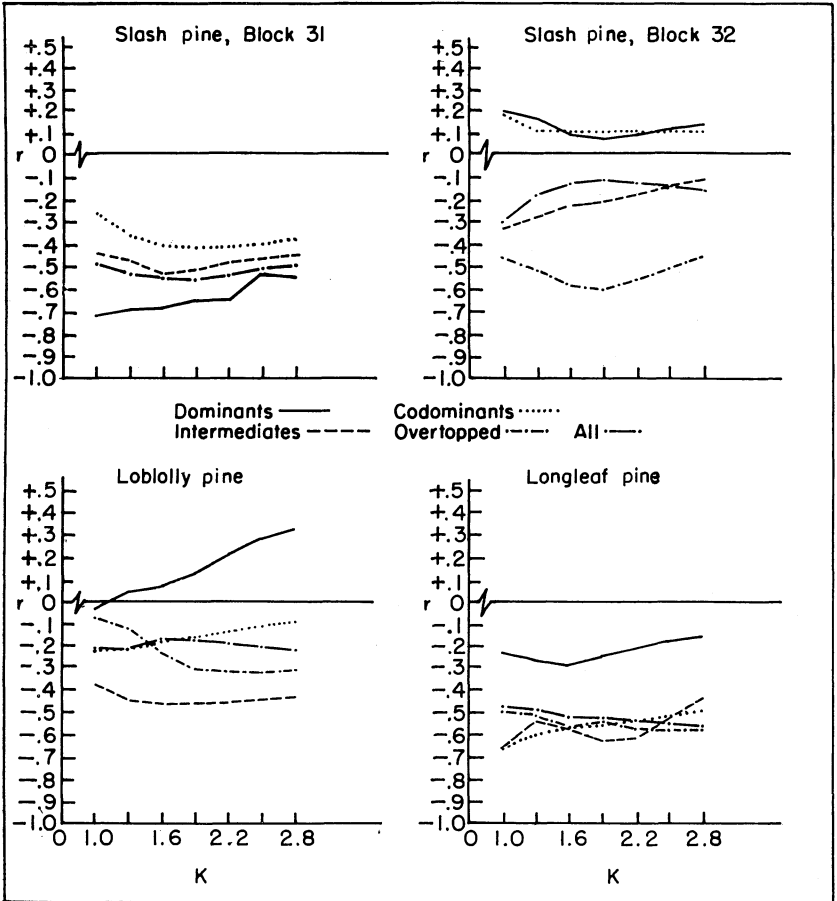


FIG. 45. Correlation coefficients associated with d.b.h. growth and the Fritts-Gerrard competition index  $I_{FG}$ .

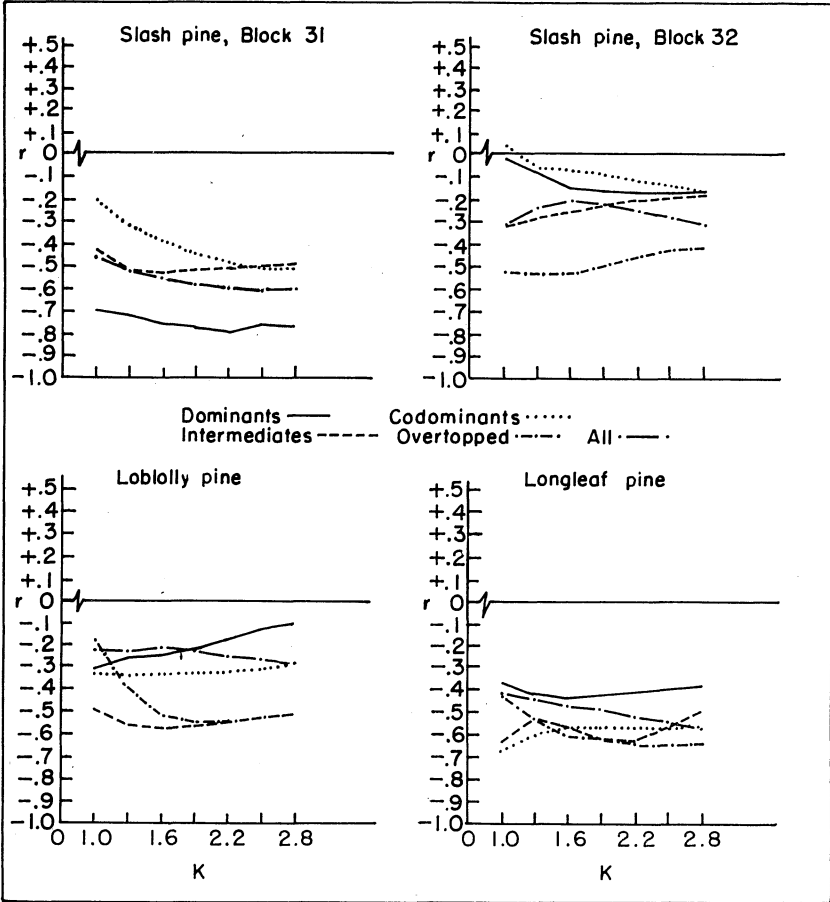


FIG. 46. Correlation coefficients associated with basal area growth and the Opie competition index  $I_0$ .

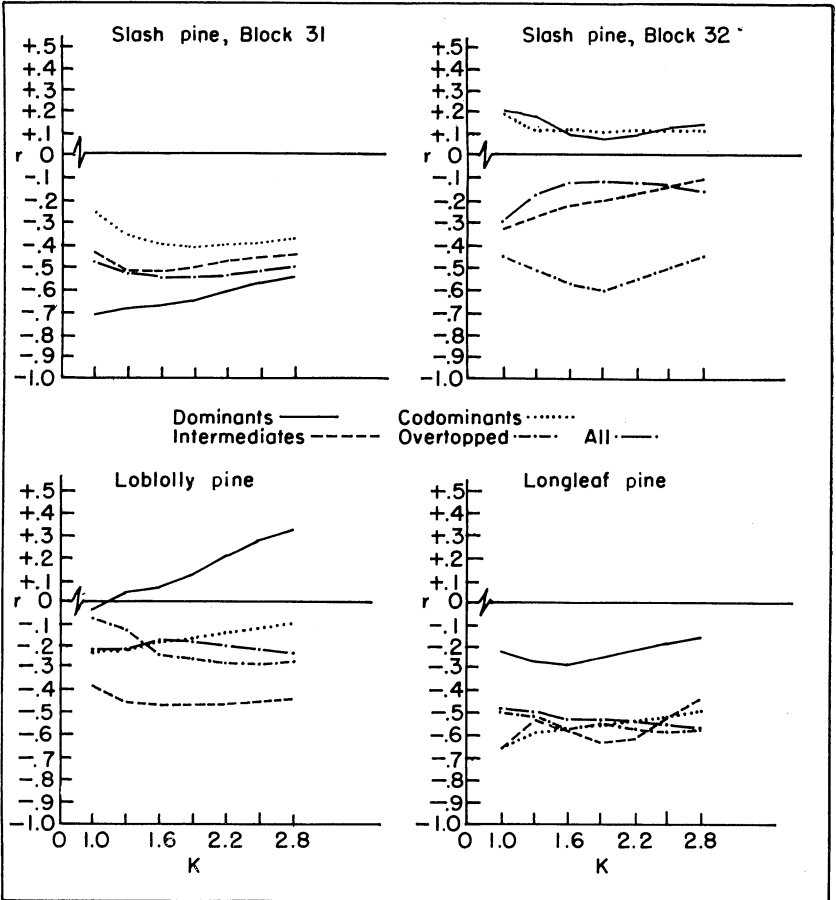


FIG. 47. Correlation coefficients associated with d.b.h. growth and the Opie competition index  $I_0$ .

**APPENDIX B**  
**Tables**

APPENDIX B, TABLE I. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING BASAL AREA PER ACRE FROM FIXED RADIUS PLOTS, WITH SAMPLE TREE

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	Large plot	33	-0.7381	Large plot	33	-0.1573	Small plot	10	-0.2986	Large plot	24	-0.3902
	√X			-0.7431**			-0.1495			-0.3099n.s.			-0.4123*
	X <sup>2</sup>			-0.6931			-0.1690n.s.			-0.2741			-0.3458
C	X	Large plot	84	-0.4449	Small plot	60	+0.0676†	Large plot	46	-0.2752	Small plot	28	-0.6238
	√X			-0.4548**			+0.0708†n.s.			-0.2489			-0.6441**
	X <sup>2</sup>			-0.4106			+0.0628†			-0.3198*			-0.5577
I	X	Large plot	39	-0.5360	Large plot	42	-0.2149	Small plot	32	-0.6649**	Large plot	12	-0.6405*
	√X			-0.5268			-0.1898			-0.6503			-0.6380
	X <sup>2</sup>			-0.5462**			-0.2572n.s.			-0.6405			-0.6402
O	X	-----	---	-----	Small plot	15	-0.3989	Large plot	10	-0.5670	Large plot	26	-0.6012
	√X			-----			-0.4049n.s.			-0.5767n.s.			-0.5862
	X <sup>2</sup>			-----			-0.3737			-0.5505			-0.6111**
All	X	Large plot	156	-0.5264	Small plot	150	-0.1734	Small plot	98	-0.2338*	Large plot	90	-0.2615
	√X			-0.5391**			-0.1570			-0.2336			-0.2802**
	X <sup>2</sup>			-0.4833			-0.1924*			-0.2238			-0.2257
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	Large plot	33	-0.6777	Small plot	33	+0.1423†	Small plot	10	-0.5720	Large plot	24	-0.3122
	√X			-0.7121**			+0.1686†n.s.			-0.5837n.s.			-0.3268n.s.
	X <sup>2</sup>			-0.6000			+0.0879†			-0.5463			-0.2784
C	X	Large plot	84	-0.4835	Small plot	60	+0.1683†	Large plot	46	-0.1739	Small plot	28	-0.5878
	√X			-0.4925**			+0.1800†n.s.			-0.1510			-0.6261**
	X <sup>2</sup>			-0.4480			+0.1489†			-0.2162n.s.			-0.5052
I	X	Large plot	39	-0.5471	Small plot	42	-0.1592	Small plot	32	-0.5625**	Large plot	12	-0.6592
	√X			-0.5392			-0.0894			-0.5391			-0.6510
	X <sup>2</sup>			-0.5556**			-0.2576n.s.			-0.5592			-0.6689*
O	X	-----	---	-----	Large plot	15	-0.5210	Large plot	10	-0.3676	Large plot	26	-0.5365
	√X			-----			-0.5152			-0.3533			-0.5251
	X <sup>2</sup>			-----			-0.5234*			-0.3975n.s.			-0.5446**
All	X	Large plot	156	-0.5614	Large plot	150	-0.1491	Small plot	98	-0.2600	Large plot	90	-0.3563
	√X			-0.5748**			-0.1355			-0.2504			-0.3686**
	X <sup>2</sup>			-0.5183			-0.1705*			-0.2676**			-0.3242

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine					
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r			
<b>P.A.I. in basal area</b>																
D	X	Large plot	33	-0.7452	Small plot	33	-0.1991	Small plot	10	-0.6533	Large plot	24	-0.3831			
	√X			-0.7527**						-0.2676n.s.				-0.6509		-0.4127*
	X <sup>2</sup>			-0.6949						-0.1678				-0.6546*		-0.3326
C	X	Large plot	84	-0.4860	Small plot	60	-0.0988	Small plot	46	-0.3857	Small plot	28	-0.6223			
	√X			-0.4914**						-0.1279n.s.				-0.3673*		-0.6970**
	X <sup>2</sup>			-0.4494						0.0988				-0.3331		-0.5050
I	X	Large plot	39	-0.5325	Large plot	42	-0.2216	Small plot	32	-0.6629**	Large plot	12	-0.6337			
	√X			-0.5220						-0.1958				-0.6186		-0.6287
	X <sup>2</sup>			-0.5439**						-0.2648n.s.				-0.6188		-0.6369*
O	X	-----	---	-----	Small plot	15	-0.4358	Large plot	10	-0.5595	Large plot	26	-0.6010			
	√X			-----						-0.4483n.s.				-0.5696n.s.		-0.5872
	X <sup>2</sup>			-----						-0.3988				-0.5430		-0.6090**
All	X	Large plot	156	-0.5740	Small plot	150	-0.3572	Small plot	98	-0.4267	Small plot	90	-0.4772			
	√X			-0.5853**						-0.3731**				-0.3646		-0.5384**
	X <sup>2</sup>			-0.5238						-0.3281				-0.4527**		-0.3610
<b>P.A.I. in d.b.h.o.b.</b>																
D	X	Large plot	33	-0.6329	Large plot	33	-0.1255	Small plot	10	-0.6401	Large plot	24	-0.2678			
	√X			-0.6674**						-0.1174				-0.6444*		-0.2788n.s.
	X <sup>2</sup>			-0.5617						-0.1377n.s.				-0.6284		-0.2387
C	X	Large plot	84	-0.4782	Large plot	60	+0.0951†	Large plot	46	-0.1896	Small plot	28	-0.5112			
	√X			-0.4812**						+0.0896†				-0.1658		-0.5715**
	X <sup>2</sup>			-0.4474						+0.1962†n.s.				-0.2323n.s.		-0.4148
I	X	Large plot	39	-0.5344	Small plot	42	-0.1624	Small plot	32	-0.5470**	Large plot	12	-0.6431			
	√X			-0.5244						-0.0844				-0.4884		-0.6314
	X <sup>2</sup>			-0.5457**						-0.2673n.s.				-0.5337		-0.6581*
O	X	-----	---	-----	Large plot	15	-0.5169	Large plot	10	-0.3512	Large plot	26	-0.5309			
	√X			-----						-0.5106				-0.3365		-0.5202
	X <sup>2</sup>			-----						-0.5206*				-0.3822n.s.		-0.5382**
All	X	Large plot	156	-0.5678	Small plot	150	-0.2320	Small plot	98	-0.4029	Small plot	90	-0.4874			
	√X			-0.5769**						-0.1963				-0.3257		-0.5186**
	X <sup>2</sup>			-0.5270						-0.2694**				-0.4483**		-0.3943

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 3. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING THE STENEKER-JARVIS COMPETITION INDEX  $I_{SJ1}$

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine				
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r		
<b>P.A.I. in basal area</b>															
D	X	Large plot	33	-0.6779	Small plot	33	-0.1437	Small plot	10	-0.2020	Large plot	24	-0.3160		
	√X			-0.7147**			n.s.			-0.2616			n.s.	-0.3778	n.s.
	X <sup>2</sup>			-0.5461			-0.0847			-0.0913			-0.2301		
C	X	Large plot	84	-0.2676	Small plot	60	-0.0107	Small plot	46	-0.3453	Small plot	28	-0.6457		
	√X			-0.3013**			n.s.			-0.3790**			-0.7214**		
	X <sup>2</sup>			-0.2066			+0.0094			-0.2747			-0.5597		
I	X	Large plot	39	-0.4043	Small plot	42	-0.0957	Small plot	32	-0.6075	Large plot	12	-0.5063 n.s.		
	√X			-0.4294**			-0.0425			-0.6243**			-0.4999		
	X <sup>2</sup>			-0.3250			-0.1398 n.s.			-0.5324			-0.4706		
O	X	-----	---	-----	Large plot	15	+0.1425	Large plot	10	-0.4669	Small plot	26	-0.3762 n.s.		
	√X			-----			+0.0982			-0.3905			-0.3760		
	X <sup>2</sup>			-----			+0.1984 n.s.			-0.5520 n.s.			-0.3685		
All	X	Small plot	156	-0.3388	Large plot	150	+0.0856 n.s.	Large plot	98	+0.0454	Large plot	90	+0.1179		
	√X			-0.4122**			+0.0779			-0.0039			+0.1022		
	X <sup>2</sup>			-0.2572			+0.0815			+0.1444 n.s.			+0.1201 n.s.		
<b>P.A.I. in d.b.h.o.b.</b>															
D	X	Large plot	33	-0.4643	Small plot	33	+0.1297	Large plot	10	+0.2181	Large plot	24	-0.1139		
	√X			-0.5139**			n.s.			+0.1778			n.s.	-0.1523 n.s.	
	X <sup>2</sup>			-0.3693			+0.0469			+0.2844 n.s.			-0.0632		
C	X	Large plot	84	-0.2404	Large plot	60	+0.2304	Small plot	46	-0.1678	Small plot	28	-0.5947		
	√X			-0.2604*			n.s.			-0.1718 n.s.			-0.6317**		
	X <sup>2</sup>			-0.1923			+0.1803			-0.1343			-0.5379		
I	X	Large plot	39	-0.3451	Small plot	42	-0.0582	Small plot	32	-0.5402	Large plot	12	-0.4361 n.s.		
	√X			-0.3545*			+0.0010			-0.5245**			-0.4119		
	X <sup>2</sup>			-0.2861			-0.1164 n.s.			-0.4991			-0.4335		
O	X	-----	---	-----	Small plot	15	-0.3162	Small plot	10	-0.5843	Large plot	26	-0.4830		
	√X			-----			-0.3686 n.s.			-0.5086			-0.5068**		
	X <sup>2</sup>			-----			-0.2362			-0.6872*			-0.4339		
All	X	Large plot	156	-0.3132	Large plot	150	+0.2037	Large plot	98	+0.0868	Small plot	90	-0.0458		
	√X			-0.3532**			+0.2176**			+0.0389			-0.1311 n.s.		
	X <sup>2</sup>			-0.2409			+0.1499			+0.1808 n.s.			+0.0110		

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

+ The sign of the correlation coefficient is reversed from what would be expected from theory.



Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	Large plot	33	-0.6572	Small plot	33	-0.1072	Small plot	10	-0.0913	Large plot	24	-0.1962
	√X			-0.7158**			-0.2183 n.s.			-0.1887 n.s.			-0.2735 n.s.
	X <sup>2</sup>			-0.5272			-0.0818			+0.0528			-0.1481
C	X	Small plot	84	-0.2072	Small plot	60	-0.0417	Small plot	46	-0.3324	Small plot	28	-0.5486
	√X			-0.2569*			-0.0940 n.s.			-0.3846**			-0.6597**
	X <sup>2</sup>			-0.1914			-0.0055			-0.2485			-0.4509
I	X	Large plot	39	-0.3878	Small plot	42	-0.1065	Small plot	32	-0.5368	Large plot	12	-0.4757
	√X			-0.4154**			-0.0579			-0.5897**			-0.4849 n.s.
	X <sup>2</sup>			-0.3045			-0.1308 n.s.			-0.4418			-0.4288
O	X	-----	---	-----	Large plot	15	+0.0674	Small plot	10	-0.4024	Large plot	26	-0.2946
	√X			-----			+0.0370			-0.3121			-0.3092 n.s.
	X <sup>2</sup>			-----			+0.0752 n.s.			-0.4961 n.s.			-0.2768
All	X	Small plot	156	-0.3488	Small plot	150	+0.0115	Small plot	98	-0.0640	Small plot	90	-0.0033
	√X			-0.4284**			-0.0981 n.s.			-0.1430 n.s.			-0.0948 n.s.
	X <sup>2</sup>			-0.2638			+0.0485			+0.0216			+0.0385
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	Large plot	33	-0.4433	Small plot	33	+0.1142	Large plot	10	+0.2424	Large plot	24	-0.0143
	√X			-0.4981**			+0.1379 n.s.			+0.1850			-0.0606 n.s.
	X <sup>2</sup>			-0.3586			+0.0124			+0.3213 n.s.			+0.0024
C	X	Large plot	84	-0.2137	Large plot	60	+0.1987	Small plot	46	-0.1728	Small plot	28	-0.5344
	√X			-0.2212*			+0.2080 n.s.			-0.1854 n.s.			-0.5986**
	X <sup>2</sup>			-0.1968			+0.1590			-0.1305			-0.4632
I	X	Large plot	39	-0.3305	Small plot	42	-0.0646	Large plot	32	-0.4789	Large plot	12	-0.4126 n.s.
	√X			-0.3423*			-0.0054			-0.4915**			-0.4015
	X <sup>2</sup>			-0.2666			-0.1088 n.s.			-0.4276			-0.3967
O	X	-----	---	-----	Small plot	15	-0.3548	Small plot	10	-0.5436	Large plot	26	-0.4026
	√X			-----			-0.4029 n.s.			-0.4743			-0.4359*
	X <sup>2</sup>			-----			-0.2938			-0.5996 n.s.			-0.3510
All	X	Large plot	156	-0.3146	Large plot	150	+0.1557	Large plot	98	+0.0339	Small plot	90	-0.0624
	√X			-0.3551**			+0.1558 n.s.			-0.0160			-0.1456 n.s.
	X <sup>2</sup>			-0.2458			+0.1130			+0.1116 n.s.			-0.0089

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 5. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING THE STENEKER-JARVIS COMPETITION INDEX  $I_{SJ3}$

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	Large plot	33	-0.6822	Small plot	33	-0.0986	Large plot	10	+0.0556	Large plot	24	-0.3331
	√X			-0.6914**			-0.2203 n.s.			-0.0053			-0.3947 n.s.
	X <sup>2</sup>			-0.6011			-0.0450			+0.1535 n.s.			-0.2441
C	X	Small plot	84	-0.0780	Large plot	60	+0.1268	Small plot	46	-0.3006	Small plot	28	-0.6312
	√X			-0.1604 n.s.			+0.1872 n.s.			-0.3380*			-0.7134**
	X <sup>2</sup>			-0.0622			+0.0558			-0.2379			-0.5414
I	X	Large plot	39	-0.4281	Small plot	42	-0.0132	Small plot	32	-0.5672	Large plot	12	-0.5304
	√X			-0.4568**			+0.0445			-0.5804**			-0.5450 n.s.
	X <sup>2</sup>			-0.3523			-0.0814 n.s.			-0.5074			-0.4684
O	X	-----	---	-----	Large plot	15	+0.4510	Large plot	10	-0.4586	Large plot	26	-0.3283
	√X			-----			+0.3408			-0.3541			-0.3114
	X <sup>2</sup>			-----			+0.5915*			-0.5930 n.s.			-0.3454 n.s.
All	X	Small plot	156	-0.1715	Large plot	150	+0.3120	Large plot	98	+0.3198	Large plot	90	+0.3672
	√X			-0.2628**			+0.3838**			+0.2970			+0.4131**
	X <sup>2</sup>			-0.1333			+0.1852			+0.3521**			+0.2845
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	Large plot	33	-0.5420	Small plot	33	+0.1262 n.s.	Large plot	10	+0.2852	Large plot	24	-0.1795
	√X			-0.5876**			+0.1140			+0.2472			-0.2130 n.s.
	X <sup>2</sup>			-0.4513			+0.0651			+0.3467 n.s.			-0.1251
C	X	Large plot	84	-0.2072	Large plot	60	+0.2861	Small plot	46	-0.1457	Large plot	28	-0.6281
	√X			-0.2333*			+0.3340**			-0.1532 n.s.			-0.6749**
	X <sup>2</sup>			-0.1769			+0.2067			-0.1075			-0.5486
I	X	Large plot	39	-0.4030	Small plot	42	-0.0289	Small plot	32	-0.5217**	Large plot	12	-0.5318 n.s.
	√X			-0.4282**			+0.0270			-0.4999			-0.5195
	X <sup>2</sup>			-0.3290			-0.0904 n.s.			-0.4922			-0.5079
O	X	-----	---	-----	Small plot	15	-0.0598	Large plot	10	-0.7087	Large plot	26	-0.4609
	√X			-----			-0.1717 n.s.			-0.6214			-0.4839*
	X <sup>2</sup>			-----			+0.0970			-0.7988**			-0.4195
All	X	Large plot	156	-0.2218	Large plot	150	+0.3669	Large plot	98	+0.3387	Large plot	90	+0.2260
	√X			-0.2624**			+0.4368**			+0.3072			+0.2532*
	X <sup>2</sup>			-0.1780			+0.2208			+0.3724**			+0.1742

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	Large plot	33	-0.6756	Small plot	33	-0.0696	Large plot	10	+0.0615	Large plot	24	-0.2031
	√X			-0.7118**			-0.1875 n.s.			-0.0241			-0.2822 n.s.
	X <sup>2</sup>			-0.5783			-0.0579			+0.1791 n.s.			-0.1411
C	X	Small plot	84	-0.0861	Large plot	60	+0.0474	Small plot	46	-0.3074	Small plot	28	-0.5216
	√X			-0.1528 n.s.			+0.0882 n.s.			-0.3533*			-0.6441**
	X <sup>2</sup>			-0.0832			+0.0157			-0.2361			-0.4188
I	X	Large plot	39	-0.4089	Small plot	42	-0.0460	Small plot	32	-0.5242	Large plot	12	-0.5075
	√X			-0.4377**			+0.0122			-0.5663**			-0.5250 n.s.
	X <sup>2</sup>			-0.3278			-0.1002 n.s.			-0.4346			-0.4479
O	X	-----	---	-----	Large plot	15	+0.4243	Large plot	10	-0.4440	Large plot	26	-0.2708
	√X			-----			+0.3163			-0.3309			-0.2710
	X <sup>2</sup>			-----			+0.5419*			-0.5722 n.s.			-0.2749 n.s.
All	X	Small plot	156	-0.2239	Large plot	150	+0.2259	Large plot	98	+0.2288	Large plot	90	+0.2445
	√X			-0.3066**			+0.2807**			+0.1955			+0.2854**
	X <sup>2</sup>			-0.1756			+0.1253			+0.2809**			+0.1837
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	Large plot	33	-0.5029	Small plot	33	+0.1150	Large plot	10	+0.2973	Large plot	24	-0.0579
	√X			-0.5623**			+0.1246 n.s.			+0.2447			-0.1024 n.s.
	X <sup>2</sup>			-0.4191			+0.0255			+0.3690 n.s.			-0.0348
C	X	Large plot	84	-0.1727	Large plot	60	+0.2339	Small plot	46	-0.1590	Small plot	28	-0.5417
	√X			-0.1935 n.s.			+0.2733 n.s.			-0.1703 n.s.			-0.6162**
	X <sup>2</sup>			-0.1622			+0.1740			-0.1167			-0.4621
I	X	Large plot	39	-0.3783	Small plot	42	-0.0442	Large plot	32	-0.4727	Large plot	12	-0.5042 n.s.
	√X			-0.4026*			+0.0145			-0.4847**			-0.4904
	X <sup>2</sup>			-0.3005			-0.0960 n.s.			-0.4254			-0.4854
O	X	-----	---	-----	Small plot	15	-0.0757	Small plot	10	-0.6732	Large plot	26	-0.4020
	√X			-----			-0.1963 n.s.			-0.5708			-0.4375*
	X <sup>2</sup>			-----			+0.0853			-0.7720**			-0.3522
All	X	Small plot	156	-0.2371	Large plot	150	+0.2979	Large plot	98	+0.2614	Large plot	90	+0.1393
	√X			-0.2910**			+0.3632**			+0.2223			+0.1570 n.s.
	X <sup>2</sup>			-0.1897			+0.1622			+0.3114**			+0.1068

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 7. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING BITTERLICH'S METHOD, WITH SAMPLE TREE

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	BAF	33	-0.6366	BAF	33	-0.1962	BAF	10	-0.5669n.s.	BAF	24	-0.4031
	√X	=25		-0.6409**	=5		-0.1912	=40		-0.5668	=10		-0.4381*
	X <sup>2</sup>			-0.6246			-0.2036n.s.			-0.5669			-0.3549
C	X	BAF	84	-0.4524	BAF	60	-0.1293	BAF	46	-0.3606	BAF	28	-0.6104
	√X	=5		-0.4796**	=15		-0.1331n.s.	=40		-0.3749**	=40		-0.6183**
	X <sup>2</sup>			-0.3837			-0.1163			-0.3215			-0.5749
I	X	BAF	39	-0.4895	BAF	42	-0.1797	BAF	32	-0.5509	BAF	12	-0.4481
	√X	=10		-0.4724	=20		-0.1260	=25		-0.5514**	=5		-0.4535n.s.
	X <sup>2</sup>			-0.5113**			-0.2627n.s.			-0.5268			-0.4356
O	X	-----	---	-----	BAF	15	-0.6400	BAF	10	-0.5191	BAF	26	-0.6516**
	√X			-----	=15		-0.6637**	=5		-0.5276n.s.	=5		-0.6446
	X <sup>2</sup>			-----			-0.5895			-0.4988			-0.6480
All	X	BAF	156	-0.5268	BAF	150	-0.2279	BAF	98	-0.3381	BAF	90	-0.3901
	√X	=5		-0.5511**	=20		-0.2007	=30		-0.3128	=10		-0.4155**
	X <sup>2</sup>			-0.4689			-0.2663**			-0.3628**			-0.3440
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	BAF	33	-0.6697	BAF	33	+0.2787†	BAF	10	-0.7035	BAF	25	-0.2848
	√X	=15		-0.6997**	=20		+0.2645†	=40		-0.7034	=15		-0.2938n.s.
	X <sup>2</sup>			-0.6020			+0.2984†n.s.			-0.7036*			-0.2431
C	X	BAF	84	-0.3987	BAF	60	+0.2141†n.s.	BAF	46	-0.3217	BAF	28	-0.6236
	√X	=15		-0.4168**	=5		+0.2129†	=40		-0.3317*	=30		-0.6460**
	X <sup>2</sup>			-0.3567			+0.2129†			-0.2833			-0.5555
I	X	BAF	39	-0.5313	BAF	42	-0.2418	BAF	32	-0.4415	BAF	12	-0.5431n.s.
	√X	=10		-0.5202	=30		-0.2099	=30		-0.4105	=40		-0.5400
	X <sup>2</sup>			-0.5422**			-0.2871n.s.			-0.4702**			-0.5263
O	X	-----	---	-----	BAF	15	-0.6237	BAF	10	-0.4371	BAF	26	-0.6130**
	√X			-----	=15		-0.6447**	=5		-0.4360	=5		-0.6058
	X <sup>2</sup>			-----			-0.5779			-0.4383n.s.			-0.6121
All	X	BAF	156	-0.5349	BAF	150	-0.2903	BAF	98	-0.3507	BAF	90	-0.4345
	√X	=15		-0.5468**	=40		-0.2612	=30		-0.3232	=10		-0.4495**
	X <sup>2</sup>			-0.4968			-0.3324**			-0.3802**			-0.3976

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	BAF	33	-0.7778	BAF	33	+0.1782†	BAF	10	-0.5669n.s.	BAF	24	-0.4109
	√X	=5		-0.7841**	=30		+0.1940†n.s.	=40		-0.5668	=10		-0.4898*
	X <sup>2</sup>			-0.7145			+0.1581†			-0.5669			-0.3498
C	X	BAF	84	-0.4533	BAF	60	-0.1293	BAF	46	-0.3606	BAF	28	-0.6104**
	√X	=5		-0.4816**	=15		-0.1334n.s.	=40		-0.3940**	=40		-0.6100
	X <sup>2</sup>			-0.3810			-0.1139			-0.2980			-0.5367
I	X	BAF	39	-0.4895	BAF	42	-0.1797	BAF	32	-0.5509**	BAF	12	-0.4347
	√X	=10		-0.4705	=20		-0.1116	=25		-0.5495	=40		-0.3770
	X <sup>2</sup>			-0.5127**			-0.2754n.s.			-0.5185			-0.4497n.s.
O	X	-----	---	-----	BAF	15	-0.6400	BAF	10	-0.5191	BAF	26	-0.6388
	√X			-----	=15		-0.6668**	=5		-0.5280n.s.	=10		-0.6487**
	X <sup>2</sup>			-----			-0.5826			-0.4977			-0.6057
All	X	BAF	156	-0.5283	BAF	150	-0.2971	BAF	98	-0.3381	BAF	90	-0.3909
	√X	=5		-0.5541**	=40		-0.2358	=30		-0.2948	=10		-0.4319**
	X <sup>2</sup>			-0.4670			-0.3400**			-0.3657**			-0.3391
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	BAF	33	-0.6697	BAF	33	+0.2787†	BAF	10	-0.7035	BAF	24	-0.2610
	√X	=15		-0.7067**	=20		+0.2598†	=40		-0.7035	=10		-0.3250n.s.
	X <sup>2</sup>			-0.5868			+0.3021†n.s.			-0.7036*			-0.2147
C	X	BAF	84	-0.3987	BAF	60	+0.2080†n.s.	BAF	46	-0.3217	BAF	28	-0.6236
	√X	=15		-0.4266**	=5		+0.2072†	=40		-0.3310*	=30		-0.6384**
	X <sup>2</sup>			-0.3483			+0.2062†			-0.2608			-0.5174
I	X	BAF	39	-0.5313	BAF	42	-0.2187	BAF	32	-0.4415	BAF	12	-0.5431n.s.
	√X	=10		-0.5190	=40		-0.1000	=30		-0.3965	=40		-0.5292
	X <sup>2</sup>			-0.5426**			-0.3052*			-0.4730**			-0.5116
O	X	-----	---	-----	BAF	15	-0.6237	BAF	10	-0.4371	BAF	26	-0.6082**
	√X			-----	=15		-0.6474**	=5		-0.4359	=5		-0.6030
	X <sup>2</sup>			-----			-0.5716			-0.4382n.s.			-0.5970
All	X	BAF	156	-0.5350	BAF	150	-0.2903	BAF	98	-0.3507	BAF	90	-0.4435
	√X	=15		-0.5483**	=40		-0.2158	=30		-0.3035	=5		-0.4636**
	X <sup>2</sup>			-0.4884			-0.3471**			-0.3843**			-0.3890

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 9. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING SPURR'S METHOD, WITH SAMPLE TREE

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	12	33	-0.7387	16	30	+0.0754†	10	10	-0.5866	12	20	-0.3639
	√X	trees		-0.7391**	trees		+0.0872†n.s.	trees		-0.5901n.s.	trees		-0.4210n.s.
	X <sup>2</sup>			-0.6863			+0.0537†			-0.5791			-0.2638
C	X	12	80	-0.1739	4	60	-0.0553	4	46	-0.3283	16	14	-0.7510
	√X	trees		-0.2346*	trees		-0.0528	trees		-0.3387*	trees		-0.7924**
	X <sup>2</sup>			-0.0645			-0.0573 n.s.			-0.2952			-0.6706
I	X	16	36	-0.5436**	4	42	-0.2178	14	28	-0.5204	14	10	-0.7807
	√X	trees		-0.5414	trees		-0.1736	trees		-0.5420**	trees		-0.7828**
	X <sup>2</sup>			-0.5400			-0.2850n.s.			-0.4743			-0.7714
O	X	-----	---	-----	8	15	-0.5464	16	10	-0.4498	14	22	-0.4726*
	√X				trees		-0.5606*	trees		-0.4657 n.s.	trees		-0.4703
	X <sup>2</sup>						-0.5022			-0.4212			-0.4606
All	X	12	151	-0.4874	10	149	-0.3569	14	90	-0.3009	4	90	-0.5369
	√X	trees		-0.5231**	trees		-0.3384	trees		-0.2958	trees		-0.5603**
	X <sup>2</sup>			-0.4051			-0.3800**			-0.3016**			-0.4640
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	14	29	-0.7506	10	33	+0.1850†	10	10	-0.3474	12	20	-0.2931
	√X	trees		-0.7856**	trees		+0.1893†n.s.	trees		-0.3476n.s.	trees		-0.3556n.s.
	X <sup>2</sup>			-0.6541			+0.1780†			-0.3462			-0.1865
C	X	16	65	-0.3061	16	55	+0.1220†	4	46	-0.2636	16	14	-0.6445
	√X	trees		-0.3341**	trees		+0.1230†n.s.	trees		-0.2781n.s.	trees		-0.6624**
	X <sup>2</sup>			-0.2499			+0.1217†			-0.2242			-0.6059
I	X	16	36	-0.5851	4	42	-0.2504	14	28	-0.4206	12	11	-0.8220
	√X	trees		-0.5854**	trees		-0.2084	trees		-0.4302*	trees		-0.8327**
	X <sup>2</sup>			-0.5774			-0.3153*			-0.3988			-0.7980
O	X	-----	---	-----	16	14	-0.5575	16	10	-0.1939	14	22	-0.4775
	√X				trees		-0.5599*	trees		-0.1988n.s.	trees		-0.4795*
	X <sup>2</sup>						-0.5357			-0.1863			-0.4574
All	X	14	139	-0.5153	8	150	-0.3446	14	90	-0.2682	4	90	-0.5606
	√X	trees		-0.5429**	trees		-0.3219	trees		-0.2604	trees		-0.5839**
	X <sup>2</sup>			-0.4578			-0.3742**			-0.2762**			-0.4753

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

†The sign of the correlation coefficient is reversed from what would be expected from theory.

TABLE 10. FIGURES: CORRELATION COEFFICIENTS OBTAINED, USING SPUR'S METHOD, WITHOUT SKINNE TABLE

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	12	33	-0.7336**	16	30	+0.1008†	8	10	-0.6873	12	20	-0.3645
	√X	trees		-0.7265	trees		+0.1110† n.s.	trees		-0.6779	trees		-0.4160 n.s.
	X <sup>2</sup>			-0.6931			+0.0825†			-0.7045*			-0.2682
C	X	12	80	-0.2154	16	55	+0.0555†	4	46	-0.3169	16	14	-0.7506
	√X	trees		-0.2672*	trees		+0.0606† n.s.	trees		-0.3264*	trees		-0.7930**
	X <sup>2</sup>			-0.1214			+0.0466†			-0.2871			-0.6715
I	X	16	36	-0.5621**	4	42	-0.2450	14	28	-0.5340	14	10	-0.7505
	√X	trees		-0.5591	trees		-0.1972	trees		-0.5566**	trees		-0.7427
	X <sup>2</sup>			-0.5600			-0.3221*			-0.4854			-0.7606*
O	X	-----	---	-----	8	15	-0.5475	16	10	-0.4752	14	22	-0.5475**
	√X				trees		-0.5591*	trees		-0.4887 n.s.	trees		-0.5443
	X <sup>2</sup>						-0.5096			-0.4515			-0.5321
All	X	12	151	-0.5044	8	150	-0.3481	8	98	-0.3204**	4	90	-0.5225
	√X	trees		-0.5344**	trees		-0.3292	trees		-0.3194	trees		-0.5390**
	X <sup>2</sup>			-0.4321			-0.3707**			-0.3075			-0.4740
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	14	29	-0.7578	8	33	+0.2298†	8	10	-0.4481	12	20	-0.2885
	√X	trees		-0.7845**	trees		+0.2272†	trees		-0.4351	trees		-0.3464 n.s.
	X <sup>2</sup>			-0.6708			+0.2349† n.s.			-0.4733 n.s.			-0.1872
C	X	16	65	-0.3493	16	55	+0.1368†	4	46	-0.2588	16	14	-0.6482
	√X	trees		-0.3726**	trees		+0.1383† n.s.	trees		-0.2734 n.s.	trees		-0.6677**
	X <sup>2</sup>			-0.3017			+0.1351†			-0.2214			-0.6089
I	X	16	36	-0.5996**	4	42	-0.2867	14	28	-0.4446	12	11	-0.7864
	√X	trees		-0.5992	trees		-0.2404	trees		-0.4563*	trees		-0.7893**
	X <sup>2</sup>			-0.5935			-0.3619*			-0.4183			-0.7751
O	X	-----	---	-----	16	14	-0.5842*	16	10	-0.2278	14	22	-0.5232
	√X				trees		-0.5827	trees		-0.2330 n.s.	trees		-0.5243*
	X <sup>2</sup>						-0.5691			-0.2203			-0.5005
All	X	14	139	-0.5350	4	150	-0.3615	8	98	-0.2954**	4	90	-0.5530
	√X	trees		-0.5582**	trees		-0.3414	trees		-0.2935	trees		-0.5667**
	X <sup>2</sup>			-0.4847			-0.3764**			-0.2860			-0.5014

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 11. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING BROWN'S METHOD

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	-----	33	+0.5795	-----	33	+0.2543	-----	10	+0.4108	-----	24	+0.3279
	√X			+0.6481**			+0.2317			+0.3735			+0.2849
	X <sup>2</sup>			+0.4743			+0.2819 n.s.			+0.4440 n.s.			+0.3629 n.s.
C	X	-----	84	+0.3297	-----	60	+0.2315 n.s.	-----	46	+0.3671	-----	28	+0.6346
	√X			+0.3430**			+0.2290			+0.3932**			+0.6430**
	X <sup>2</sup>			+0.2838			+0.2072			+0.2861			+0.5801
I	X	-----	39	+0.2116	-----	42	+0.0016	-----	32	+0.5279	-----	12	+0.2892
	√X			+0.2736 n.s.			+0.0530			+0.5340**			+0.3329 n.s.
	X <sup>2</sup>			+0.0828			-0.0880 † n.s.			+0.4902			+0.1629
O	X	-----	---	-----	-----	15	+0.1655	-----	10	+0.1680	-----	26	+0.4801
	√X			-----			+0.2134 n.s.			+0.1710 n.s.			+0.5141**
	X <sup>2</sup>			-----			+0.1030			+0.1586			+0.3990
All	X	-----	156	+0.4534	-----	150	+0.3351	-----	98	+0.3474	-----	90	+0.4492
	√X			+0.4952**			+0.3368**			+0.3610**			+0.4571**
	X <sup>2</sup>			+0.3530			+0.3196			+0.2905			+0.3732
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	-----	33	+0.4675	-----	33	-0.1800 †	-----	10	-0.0314 †	-----	24	+0.0835
	√X			+0.4765**			-0.2016 † n.s.			-0.0627 † n.s.			+0.0405
	X <sup>2</sup>			+0.4547			-0.1253 †			+0.0104			+0.1370 n.s.
C	X	-----	84	+0.1958 n.s.	-----	60	-0.0844 †	-----	46	+0.1760	-----	28	+0.5124
	√X			+0.1949			-0.0886 † n.s.			+0.2007 n.s.			+0.5159**
	X <sup>2</sup>			+0.1861			-0.0863 †			+0.1107			+0.4818
I	X	-----	39	+0.0796	-----	42	-0.0971 †	-----	32	+0.3909	-----	12	+0.1698
	√X			+0.1411 n.s.			-0.0453 †			+0.4027*			+0.2072 n.s.
	X <sup>2</sup>			-0.0251 †			-0.1816 † n.s.			+0.3524			+0.0592
O	X	-----	---	-----	-----	15	+0.2313	-----	10	+0.1562	-----	26	+0.4401
	√X			-----			+0.2789 n.s.			+0.1449			+0.4676*
	X <sup>2</sup>			-----			+0.1668			+0.1773 n.s.			+0.3739
All	X	-----	156	+0.3330	-----	150	+0.1125	-----	98	+0.2656	-----	90	+0.3569
	√X			+0.3569**			+0.1296 n.s.			+0.2805**			+0.3902**
	X <sup>2</sup>			+0.2781			+0.0907			+0.2162			+0.2559

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.



Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	-----	33	+0.6210	-----	33	+0.2365 n.s.	-----	10	+0.4821	-----	24	+0.3591
	√X			+0.6765**			+0.2342			+0.4616			+0.3195
	X <sup>2</sup>			+0.5178			+0.2344			+0.4870 n.s.			+0.4055*
C	X	-----	84	+0.3559	-----	60	+0.3837	-----	46	+0.4286	-----	28	+0.5828
	√X			+0.3810**			+0.4079**			+0.4372**			+0.5890**
	X <sup>2</sup>			+0.2889			+0.3137			+0.3752			+0.5430
I	X	-----	39	+0.1846	-----	42	+0.0674	-----	32	+0.5675	-----	12	+0.2044
	√X			+0.2123 n.s.			+0.1250 n.s.			+0.5760**			+0.2441 n.s.
	X <sup>2</sup>			+0.1406			-0.0400†			+0.5108			+0.1197
O	X	-----	---	-----	-----	15	+0.3441	-----	10	+0.0383	-----	26	+0.5227**
	√X			-----			+0.4135 n.s.			-0.0067†			+0.5201
	X <sup>2</sup>			-----			+0.1701			+0.0709 n.s.			+0.4699
All	X	-----	156	+0.5107	-----	150	+0.5336	-----	98	+0.5048	-----	90	+0.6474
	√X			+0.5655**			+0.5701**			+0.5245**			+0.7455**
	X <sup>2</sup>			+0.3693			+0.4412			+0.4417			+0.4361
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	-----	33	+0.4773	-----	33	-0.2238†	-----	10	+0.0592	-----	24	+0.1284
	√X			+0.4772			-0.2302† n.s.			+0.0443			+0.0860
	X <sup>2</sup>			+0.4798**			-0.1965†			+0.0705 n.s.			+0.1896 n.s.
C	X	-----	84	+0.2021 n.s.	-----	60	+0.0538	-----	46	+0.2105	-----	28	+0.4236
	√X			+0.2018			+0.0810 n.s.			+0.2221 n.s.			+0.4276*
	X <sup>2</sup>			+0.1882			+0.0175			+0.1679			+0.3989
I	X	-----	39	+0.0515	-----	42	-0.0440†	-----	32	+0.4259	-----	12	+0.0900
	√X			+0.0852 n.s.			+0.0123			+0.4419*			+0.1253 n.s.
	X <sup>2</sup>			+0.0090			-0.1421† n.s.			+0.3664			+0.0165
O	X	-----	---	-----	-----	15	+0.0819	-----	10	-0.3250	-----	26	+0.3044
	√X			-----			+0.1283 n.s.			-0.3914† n.s.			+0.3076 n.s.
	X <sup>2</sup>			-----			-0.0375†			-0.2438			+0.2593
All	X	-----	156	+0.3640	-----	150	+0.2932	-----	98	+0.4023	-----	90	+0.4783
	√X			+0.3929**			+0.3619**			+0.4346**			+0.5878**
	X <sup>2</sup>			+0.2864			+0.1968			+0.3270			+0.2855

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 13. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING STAEBLER'S COMPETITION INDEX  $I_{S1}$

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=2.8	33	-0.7390	K=1.0	33	+0.1847†	K=1.0	10	+0.2052†	K=1.3	24	-0.3872
	√X			-0.7665**			+0.1979†n.s.			+0.1856†			-0.4427*
	X <sup>2</sup>			-0.6517			+0.1567†			+0.2420†n.s.			-0.3022
C	X	K=2.8	84	-0.2793	K=1.0	60	+0.3112†	K=1.0	46	-0.1595	K=1.0	28	-0.6179
	√X			-0.2874**			+0.3552†**			-0.1547			-0.6271**
	X <sup>2</sup>			-0.2592			+0.2288†			-0.1737n.s.			-0.5588
I	X	K=1.3	39	-0.5262**	K=1.0	42	-0.0642	K=1.3	32	-0.5365	K=1.0	12	-0.6109
	√X			-0.5226			+0.0153†			-0.5473**			-0.5986
	X <sup>2</sup>			-0.5213			-0.1689n.s.			-0.5116			-0.6158*
O	X	-----	---	-----	K=2.8	15	+0.2756†	K=1.6	10	-0.5772	K=1.6	26	-0.5761**
	√X			-----			+0.2326†			-0.5503			-0.5514
	X <sup>2</sup>			-----			+0.3518†n.s.			-0.6202n.s.			-0.5418
All	X	K=2.8	156	-0.3045	K=1.6	150	+0.4869†	K=1.6	98	+0.5025†**	K=1.9	90	+0.3483†
	√X			-0.3192**			+0.5241†**			+0.4984†			+0.3853†**
	X <sup>2</sup>			-0.2697			+0.3886†			+0.4829†			+0.2639†
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.7285	K=1.0	33	+0.1481†	K=2.8	10	+0.4513†	K=1.6	24	-0.3216
	√X			-0.7633**			+0.1492†n.s.			+0.4408†			-0.3617n.s.
	X <sup>2</sup>			-0.6383			+0.1456†			+0.4562†n.s.			-0.2357
C	X	K=2.2	84	-0.3257	K=2.2	60	+0.2996†	K=1.0	46	-0.1324	K=1.0	28	-0.6734
	√X			-0.3549**			+0.3324†**			-0.1382n.s.			-0.7076**
	X <sup>2</sup>			-0.2781			+0.2385†			-0.1264			-0.5844
I	X	K=1.3	39	-0.5775	K=1.0	42	-0.1352	K=1.3	32	-0.4853	K=1.0	12	-0.6638
	√X			-0.5801**			-0.0600			-0.4903**			-0.6447
	X <sup>2</sup>			-0.5604			-0.2297n.s.			-0.4708			-0.6778*
O	X	-----	---	-----	K=1.0	15	-0.4132	K=1.6	10	-0.7129	K=2.2	26	-0.6282**
	√X			-----			-0.4309n.s.			-0.6899			-0.6268
	X <sup>2</sup>			-----			-0.3524			-0.7479*			-0.5802
All	X	K=2.8	156	-0.3422	K=1.9	150	+0.4334†	K=1.6	98	+0.4782†**	K=2.2	90	+0.1580†
	√X			-0.3694**			+0.4647†**			+0.4682†			+0.1757†**
	X <sup>2</sup>			-0.2894			+0.3516†			+0.4668†			+0.1218†

\* Significant at 0.05.  
 \*\* Significant at 0.01.  
 n.s. Not significant.

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=1.9	33	-0.6905	K=1.9	33	-0.1948	K=1.0	10	-0.1209	K=1.3	24	-0.3719
	√X			-0.7327**			-0.2362 n.s.			-0.1806 n.s.			-0.4313*
	X <sup>2</sup>			-0.5692			-0.1214			-0.0154			-0.2807
C	X	K=2.5	84	-0.3752	K=2.8	60	-0.1115	K=1.0	46	-0.3398	K=1.0	28	-0.6452
	√X			-0.4229**			-0.1578 n.s.			-0.3558*			-0.6802**
	X <sup>2</sup>			-0.2585			-0.0456			-0.2956			-0.5547
I	X	K=1.3	39	-0.4443	K=1.0	42	-0.1568	K=1.3	32	-0.5730	K=1.0	12	-0.5867
	√X			-0.4659**			-0.0933			-0.5991**			-0.5565
	X <sup>2</sup>			-0.3714			-0.1981 n.s.			-0.5046			-0.6067*
O	X	-----	---	-----	K=1.0	15	-0.3877 n.s.	K=1.6	10	-0.4439	K=2.2	26	-0.5808
	√X			-----			-0.3551			-0.4596 n.s.			-0.5881**
	X <sup>2</sup>			-----			-0.3590			-0.4156			-0.5376
All	X	K=2.5	156	-0.4437	K=2.8	150	-0.1466	K=2.5	98	-0.1280	K=2.8	90	-0.3391
	√X			-0.4995**			-0.1949*			-0.1708 n.s.			-0.3676**
	X <sup>2</sup>			-0.3257			-0.0654			-0.0389			-0.2840
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.5063	K=1.0	33	+0.1877†	K=2.8	10	+0.3372†	K=1.3	24	-0.1769
	√X			-0.5842**			+0.2087† n.s.			+0.3058†			-0.2179 n.s.
	X <sup>2</sup>			-0.3954			+0.1173†			+0.3592† n.s.			-0.1194
C	X	K=1.3	84	-0.2656	K=1.0	60	+0.1945†	K=1.0	46	-0.1810	K=1.0	28	-0.5769
	√X			-0.2993**			+0.2154† n.s.			-0.1917 n.s.			-0.6227**
	X <sup>2</sup>			-0.2044			+0.1580†			-0.1570			-0.4928
I	X	K=1.3	39	-0.3885	K=1.0	42	-0.1239	K=1.3	32	-0.4954	K=1.0	12	-0.5533
	√X			-0.3968*			-0.0563			-0.5034**			-0.5048
	X <sup>2</sup>			-0.3334			-0.1760 n.s.			-0.4585			-0.6035*
O	X	-----	---	-----	K=1.3	15	-0.5438	K=1.9	10	-0.4001	K=2.2	26	-0.5983
	√X			-----			-0.5587*			-0.3959			-0.6151**
	X <sup>2</sup>			-----			-0.4736			-0.4063 n.s.			-0.5440
All	X	K=1.0	156	-0.3845	K=1.9	150	+0.0428†	K=1.0	98	-0.0987	K=2.8	90	-0.3567
	√X			-0.4239**			+0.0367†			-0.1131 n.s.			-0.3762**
	X <sup>2</sup>			-0.3144			+0.0521† n.s.			-0.0772			-0.3109

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 15. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING STAEBLER'S COMPETITION INDEX  $I_{S2}/F$

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=2.5	33	-0.7263	K=2.8	33	-0.1228	K=1.0	10	-0.0505	K=1.6	24	-0.3656
	√X			-0.7565**			-0.1474 n.s.			-0.1099 n.s.			-0.4390*
	X <sup>2</sup>			-0.6341			-0.0731			+0.0524†			-0.2519
C	X	K=2.8	84	-0.3206	K=1.0	60	+0.0498†	K=1.0	46	-0.2964	K=1.0	28	-0.6657
	√X			-0.3475**			+0.1053† n.s.			-0.3085*			-0.7119**
	X <sup>2</sup>			-0.2613			+0.0064†			-0.2486			-0.5478
I	X	K=1.6	39	-0.4987	K=1.0	42	-0.1700	K=1.6	32	-0.5472	K=1.0	12	-0.6600
	√X			-0.5123**			-0.0795			-0.5629**			-0.6624*
	X <sup>2</sup>			-0.4503			-0.2597 n.s.			-0.5035			-0.6056
O	X	-----	---	-----	K=1.0	15	-0.3006	K=2.5	10	-0.4944	K=2.2	26	-0.5692**
	√X			-----			-0.2647			-0.4649			-0.5603
	X <sup>2</sup>			-----			-0.3223 n.s.			-0.5347 n.s.			-0.5270
All	X	K=2.8	156	-0.3988	K=1.9	150	+0.1255†	K=2.2	98	+0.1406†	K=1.0	90	-0.1307
	√X			-0.4338**			+0.1598† n.s.			+0.1203†			-0.1321 n.s.
	X <sup>2</sup>			-0.3264			+0.0777†			+0.1895† n.s.			-0.0593
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.5777	K=1.0	33	+0.1613†	K=2.8	10	+0.3136	K=1.6	24	-0.2231
	√X			-0.6796**			+0.1728† n.s.			+0.2904			-0.2797 n.s.
	X <sup>2</sup>			-0.4532			+0.1276†			+0.3466 n.s.			-0.1314
C	X	K=2.5	84	-0.2884	K=1.0	60	+0.1950†	K=1.0	46	-0.1825	K=1.0	28	-0.6350
	√X			-0.3109**			+0.2335† n.s.			-0.1975 n.s.			-0.6998**
	X <sup>2</sup>			-0.2425			+0.1495†			-0.1387			-0.5193
I	X	K=1.6	39	-0.4815	K=1.0	42	-0.1725	K=1.6	32	-0.4667	K=1.0	12	-0.6987*
	√X			-0.4931**			-0.0872			-0.4700**			-0.6757
	X <sup>2</sup>			-0.4338			-0.2612 n.s.			-0.4463			-0.6717
O	X	-----	---	-----	K=1.0	15	-0.5946*	K=2.5	10	-0.5582	K=2.5	26	-0.6154
	√X			-----			-0.5617			-0.5301			-0.6369**
	X <sup>2</sup>			-----			-0.5639			-0.5988 n.s.			-0.5440
All	X	K=1.6	156	-0.3774	K=2.2	150	+0.2153†	K=2.2	98	+0.1797†	K=1.0	90	-0.2276
	√X			-0.4131**			+0.2431†**			+0.1576†			-0.2547*
	X <sup>2</sup>			-0.3145			+0.1695†			+0.2273†*			-0.1189

\* Significant at 0.05.  
 \*\* Significant at 0.01.  
 n.s. Not significant.

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=1.9	33	-0.7492	K=1.9	33	-0.1512	K=1.6	10	-0.1834	K=1.6	24	-0.3876
	√X			-0.7728**			-0.1602 n.s.			-0.2057 n.s.			-0.4485*
	X <sup>2</sup>			-0.6688			-0.1275			-0.1369			-0.2804
C	X	K=2.8	84	-0.4050	K=2.8	60	-0.1066	K=1.0	46	-0.3136	K=1.0	28	-0.6616**
	√X			-0.4379**			-0.1233 n.s.			-0.3167*			-0.6607
	X <sup>2</sup>			-0.3243			-0.0844			-0.2973			-0.6051
I	X	K=1.3	39	-0.5393	K=1.0	42	-0.1879	K=1.3	32	-0.5275	K=1.0	12	-0.6514*
	√X			-0.5413**			-0.1050			-0.5456**			-0.6417
	X <sup>2</sup>			-0.5155			-0.2938 n.s.			-0.4837			-0.6365
O	X	-----	---	-----	K=1.3	15	-0.4743	K=1.9	10	-0.4960	K=2.2	26	-0.6222**
	√X			-----			-0.4586			-0.5248 n.s.			-0.6089
	X <sup>2</sup>			-----			-0.4794 n.s.			-0.4443			-0.6055
All	X	K=2.5	156	-0.4982	K=2.8	150	-0.1165	K=2.8	98	-0.1441	K=2.8	90	-0.4172
	√X			-0.5380**			-0.1512 n.s.			-0.1643 n.s.			-0.4406**
	X <sup>2</sup>			-0.4128			-0.0570			-0.0984			-0.3664
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.6475	K=1.0	33	+0.1914†	K=2.8	10	+0.3411†	K=1.6	24	-0.2370
	√X			-0.7109**			+0.1996† n.s.			+0.3175†			-0.2882 n.s.
	X <sup>2</sup>			-0.5332			+0.1740†			+0.3691† n.s.			-0.1495
C	X	K=1.9	84	-0.3270	K=1.0	60	+0.1070†	K=1.0	46	-0.2085	K=1.0	28	-0.6271
	√X			-0.3497**			+0.1536† n.s.			-0.2188 n.s.			-0.6505**
	X <sup>2</sup>			-0.2800			+0.0481†			-0.1896			-0.5576
I	X	K=1.3	39	-0.5373	K=1.0	42	-0.1933	K=1.3	32	-0.4308	K=1.0	12	-0.6790*
	√X			-0.5389**			-0.1144			-0.4362*			-0.6581
	X <sup>2</sup>			-0.5118			-0.2985 n.s.			-0.4124			-0.6781
O	X	-----	---	-----	K=1.6	15	-0.6108	K=2.2	10	-0.3302	K=2.5	26	-0.6223
	√X			-----			-0.6114*			-0.3342 n.s.			-0.6269**
	X <sup>2</sup>			-----			-0.6017			-0.3239			-0.5864
All	X	K=1.3	156	-0.4649	K=1.0	150	-0.0584 n.s.	K=2.8	98	-0.0746	K=2.8	90	-0.4515
	√X			-0.4898**			-0.0317			-0.0871 n.s.			-0.4664**
	X <sup>2</sup>			-0.4106			-0.0502			-0.0442			-0.4118

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 17. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING STAEBLER'S COMPETITION INDEX  $I_{84}/F$

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=2.8	33	-0.7442	K=1.6	33	+0.1016†	K=1.3	10	+0.0667†	K=1.6	24	-0.3585
	√X			-0.7665**			+0.1250†n.s.			+0.0518†			-0.4400*
	X <sup>2</sup>			-0.6645			+0.0562†			+0.0971†n.s.			-0.2330
C	X	K=2.8	84	-0.2440	K=1.0	60	+0.0326†	K=1.0	46	-0.2205	K=1.0	28	-0.6794
	√X			-0.2587*			+0.1238†n.s.			-0.2298n.s.			-0.6836**
	X <sup>2</sup>			-0.2230			-0.0673			-0.1790			-0.6001
I	X	K=2.2	39	-0.5624	K=1.0	42	-0.2010	K=1.6	32	-0.4022	K=1.0	12	-0.6676*
	√X			-0.5508			-0.0936			-0.4131*			-0.6648
	X <sup>2</sup>			-0.5733**			-0.3257*			-0.3815			-0.6381
O	X	-----	---	-----	K=1.0	15	-0.3653	K=2.5	10	-0.5549	K=2.5	26	-0.5900**
	√X			-----			-0.3093			-0.5508			-0.5737
	X <sup>2</sup>			-----			-0.3797n.s.			-0.5619n.s.			-0.5581
All	X	K=2.8	156	-0.3675	K=2.5	150	+0.1116†	K=2.2	98	+0.3328†	K=1.0	90	-0.1853n.s.
	√X			-0.3861**			+0.2015†*			+0.3289†			-0.1693
	X <sup>2</sup>			-0.3266			+0.0176†			+0.3377†**			-0.1111
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.9	33	-0.7316	K=1.0	33	+0.0854†	K=2.8	10	+0.3024†	K=1.6	24	-0.2706
	√X			-0.7743**			+0.1131†n.s.			+0.2944†			-0.3337n.s.
	X <sup>2</sup>			-0.6290			+0.0385†			+0.3187†n.s.			-0.1591
C	X	K=2.8	84	-0.3245	K=1.0	60	+0.0943†	K=1.0	46	-0.1922	K=1.0	28	-0.6827
	√X			-0.3490**			+0.1575†n.s.			-0.2081n.s.			-0.7142**
	X <sup>2</sup>			-0.2812			+0.0180†			-0.1373			-0.5918
I	X	K=1.9	39	-0.6243	K=1.0	42	-0.2423	K=2.8	32	-0.3236	K=1.0	12	-0.7286**
	√X			-0.6181			-0.1423			-0.3179			-0.7198
	X <sup>2</sup>			-0.6267**			-0.3619*			-0.3289n.s.			-0.7073
O	X	-----	---	-----	K=1.0	15	-0.6107*	K=2.8	10	-0.5680	K=2.5	26	-0.6343
	√X			-----			-0.5583			-0.5539			-0.6426**
	X <sup>2</sup>			-----			-0.5874			-0.5925n.s.			-0.5709
All	X	K=2.8	156	-0.4335	K=2.8	150	+0.1544†	K=2.2	98	+0.3337†	K=1.0	90	-0.2978
	√X			-0.4620**			+0.2220†**			+0.3259†			-0.3052**
	X <sup>2</sup>			-0.3754			+0.0729†			+0.3446†**			-0.1846

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=2.2	33	-0.7542	K=2.2	33	-0.1404	K=2.8	10	+0.0391†	K=1.3	24	-0.3764
	√X			-0.7774**			-0.1574 n.s.			-0.0121			-0.4336*
	X <sup>2</sup>			-0.6697			-0.1003			+0.1131† n.s.			-0.2850
C	X	K=2.8	84	-0.4153	K=2.8	60	-0.0913	K=1.0	46	-0.3035*	K=1.0	28	-0.6388
	√X			-0.4509**			-0.1113 n.s.			-0.3030			-0.6576**
	X <sup>2</sup>			-0.3340			-0.0566			-0.2978			-0.5695
I	X	K=1.3	39	-0.5199	K=1.0	42	-0.1499	K=1.3	32	-0.5780	K=1.0	12	-0.6008
	√X			-0.5221**			-0.0674			-0.5981**			-0.5774
	X <sup>2</sup>			-0.4969			-0.2561 n.s.			-0.5261			-0.6121*
O	X	-----	---	-----	K=1.3	15	-0.4627	K=1.9	10	-0.5870	K=2.2	26	-0.6228**
	√X			-----			-0.4354			-0.5998 n.s.			-0.6128
	X <sup>2</sup>			-----			-0.4985 n.s.			-0.5628			-0.6100
All	X	K=2.8	156	-0.4850	K=2.8	150	-0.1181	K=1.6	98	+0.0545†	K=2.8	90	-0.3143
	√X			-0.5262**			-0.1482 n.s.			+0.0275†			-0.3366**
	X <sup>2</sup>			-0.3977			-0.0541			+0.1140† n.s.			-0.2583
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.6540	K=2.8	33	+0.1289†	K=2.8	10	+0.3783†	K=1.6	24	-0.2362
	√X			-0.7137**			+0.1420† n.s.			+0.3535†			-0.2788 n.s.
	X <sup>2</sup>			-0.5418			+0.1028†			+0.4013† n.s.			-0.1576
C	X	K=1.9	84	-0.3599	K=1.0	60	+0.2253†	K=1.0	46	-0.1896	K=1.0	28	-0.6221
	√X			-0.3887**			+0.2497† n.s.			-0.1949 n.s.			-0.6625**
	X <sup>2</sup>			-0.3018			+0.1880†			-0.1822			-0.5384
I	X	K=1.3	39	-0.5055	K=1.0	42	-0.1635	K=1.3	32	-0.4904	K=1.0	12	-0.6097
	√X			-0.5067**			-0.0835			-0.4980**			-0.5738
	X <sup>2</sup>			-0.4832			-0.2681 n.s.			-0.4628			-0.6388*
O	X	-----	---	-----	K=1.6	15	-0.5909	K=1.9	10	-0.4523	K=2.2	26	-0.5856**
	√X			-----			-0.5833			-0.4459			-0.5782
	X <sup>2</sup>			-----			-0.5957*			-0.4654 n.s.			-0.5734
All	X	K=1.3	156	-0.4488	K=1.9	150	+0.0896†	K=1.6	98	+0.0905†	K=2.8	90	-0.3625
	√X			-0.4822**			+0.0912†			+0.0634†			-0.3811**
	X <sup>2</sup>			-0.3838			+0.0925† n.s.			+0.1471† n.s.			-0.3105

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 19. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING NEWNHAM'S COMPETITION INDEX  $I_N$

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=1.6	33	-0.7909	K=1.6	33	-0.2259	K=1.3	10	-0.4410	K=1.3	24	-0.3641
	$\sqrt{X}$			-0.8105**			-0.2157			-0.4337			-0.4262*
	$X^2$			-0.7216			-0.2450 n.s.			-0.4556 n.s.			-0.2882
C	X	K=2.2	84	-0.5912	K=2.8	60	-0.2473	K=1.6	46	-0.3815	K=1.0	28	-0.5690**
	$\sqrt{X}$			-0.6028**			-0.3121*			-0.3713			-0.5539
	$X^2$			-0.5444			-0.1546			-0.3946**			-0.5527
I	X	K=1.6	39	-0.5285	K=1.0	42	-0.2333	K=1.0	32	-0.4946	K=1.9	12	-0.6770
	$\sqrt{X}$			-0.5135			-0.1595			-0.5181**			-0.6670
	$X^2$			-0.5476**			-0.3364*			-0.4419			-0.6917*
O	X	-----	---	-----	K=1.6	15	-0.4795	K=1.0	10	+0.0655†	K=2.8	26	-0.4494
	$\sqrt{X}$	-----	---	-0.5302*			-0.0249			-0.4789*			
	$X^2$	-----	---	-0.3821			+0.2099† n.s.			-0.3855			
All	X	K=1.9	156	-0.7130	K=2.8	150	-0.4765	K=2.8	98	-0.4517	K=2.8	90	-0.7284
	$\sqrt{X}$			-0.7337**			-0.5420**			-0.4945**			-0.8027**
	$X^2$			-0.6432			-0.3520			-0.3462			-0.5502
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.6792	K=2.8	33	+0.1988†	K=2.8	10	+0.3142†	K=1.3	24	-0.2289
	$\sqrt{X}$			-0.7216**			+0.2031† n.s.			+0.2911†			-0.2782 n.s.
	$X^2$			-0.5966			+0.1855†			+0.3428† n.s.			-0.1676
C	X	K=1.6	84	-0.4447	K=2.8	60	-0.0597	K=1.0	46	-0.2851	K=1.0	28	-0.5200
	$\sqrt{X}$			-0.4477**			-0.0819 n.s.			-0.2831			-0.5252**
	$X^2$			-0.4303			-0.0219			-0.2923*			-0.4840
I	X	K=1.3	39	-0.5314	K=1.0	42	-0.2301	K=1.0	32	-0.3905	K=1.9	12	-0.6398
	$\sqrt{X}$			-0.5187			-0.1576			-0.3969*			-0.6251
	$X^2$			-0.5472**			-0.3364*			-0.3705			-0.6633*
O	X	-----	---	-----	K=1.3	15	-0.1308	K=1.0	10	+0.4622†	K=1.0	26	-0.3556
	$\sqrt{X}$	-----	---	-0.2173 n.s.			+0.3819†			-0.4331*			
	$X^2$	-----	---	-0.0175			+0.5775† n.s.			-0.2415			
All	X	K=1.3	156	-0.6134	K=2.8	150	-0.3813	K=1.0	98	-0.4462	K=2.8	90	-0.6385
	$\sqrt{X}$			-0.6205**			-0.4114**			-0.4550**			-0.6830**
	$X^2$			-0.5873			-0.2978			-0.3894			-0.4987

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.



Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=1.6	33	-0.7724	K=1.9	33	-0.1791	K=1.6	10	-0.1514	K=1.3	24	-0.3935
	√X			-0.7886**			-0.1863 n.s.			-0.1792 n.s.			-0.4466*
	X <sup>2</sup>			-0.6963			-0.1579			-0.0982			-0.3146
C	X	K=2.2	84	-0.4803	K=2.8	60	-0.1895	K=1.0	46	-0.3197	K=1.0	28	-0.5916**
	√X			-0.5125**			-0.2299 n.s.			-0.3138			-0.5908
	X <sup>2</sup>			-0.3980			-0.1216			-0.3280*			-0.5592
I	X	K=1.0	39	-0.5420**	K=1.0	42	-0.1450	K=1.0	32	-0.5738	K=1.9	12	-0.6715*
	√X			-0.5393			-0.0675			-0.6030**			-0.6713
	X <sup>2</sup>			-0.5332			-0.2573 n.s.			-0.5047			-0.6595
O	X	-----	---	-----	K=1.3	15	-0.5477	K=1.6	10	-0.5224	K=2.8	26	-0.6194**
	√X	-----	---	-----			-0.5349			-0.5469 n.s.			-0.6162
	X <sup>2</sup>	-----	---	-----			-0.5527*			-0.4767			-0.6020
All	X	K=2.2	156	-0.5701	K=2.8	150	-0.3337	K=2.8	98	-0.2913	K=2.8	90	-0.5971
	√X			-0.6094**			-0.3660**			-0.3026**			-0.6259**
	X <sup>2</sup>			-0.4803			-0.2588			-0.2665			-0.5297
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.6676	K=1.0	33	+0.1765†	K=2.8	10	+0.3983	K=1.3	24	-0.2616
	√X			-0.7097**			+0.1854† n.s.			+0.3699			-0.3053 n.s.
	X <sup>2</sup>			-0.5756			+0.1614†			+0.4189† n.s.			-0.1935
C	X	K=1.3	84	-0.3910	K=1.0	60	+0.2112†	K=1.0	46	-0.2074	K=1.0	28	-0.5726
	√X			-0.4217**			+0.2301† n.s.			-0.2076			-0.5935**
	X <sup>2</sup>			-0.3282			+0.1844†			-0.2142 n.s.			-0.5186
I	X	K=1.0	39	-0.5372**	K=1.0	42	-0.1537	K=1.0	32	-0.4864	K=1.6	12	-0.6543
	√X			-0.5322			-0.0767			-0.5010**			-0.6405
	X <sup>2</sup>			-0.5315			-0.2675 n.s.			-0.4450			-0.6659*
O	X	-----	---	-----	K=1.6	15	-0.5790*	K=2.2	10	-0.2935	K=2.2	26	-0.5674**
	√X	-----	---	-----			-0.5741			-0.2920			-0.5589
	X <sup>2</sup>	-----	---	-----			-0.5790			-0.3002 n.s.			-0.5577
All	X	K=1.3	156	-0.5030	K=1.0	150	-0.1295	K=2.8	98	-0.2311	K=2.8	90	-0.5814
	√X			-0.5305**			-0.0804			-0.2269			-0.5876**
	X <sup>2</sup>			-0.4394			-0.1944*			-0.2333*			-0.5505

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 21. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING THE FRITTS-GERRARD COMPETITION INDEX  $I_{FG}$

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=2.2	33	-0.7849	K=2.2	33	-0.1785	K=1.0	10	-0.2749	K=1.6	24	-0.3633
	√X			-0.8041**			-0.1815n.s.			-0.3045n.s.			-0.4253*
	X <sup>2</sup>			-0.7041			-0.1674			-0.2125			-0.2634
C	X	K=2.8	84	-0.4870	K=2.8	60	-0.1481	K=1.3	46	-0.3375*	K=1.0	28	-0.6563
	√X			-0.5176**			-0.1711n.s.			-0.3348			-0.6694**
	X <sup>2</sup>			-0.4098			-0.1061			-0.3370			-0.5833
I	X	K=1.6	39	-0.5413	K=1.0	42	-0.2066	K=1.6	32	-0.5594	K=1.0	12	-0.6290*
	√X			-0.5332			-0.1122			-0.5767**			-0.6178
	X <sup>2</sup>			-0.5449**			-0.3298*			-0.5169			-0.6058
O	X	-----	---	-----	K=1.3	15	-0.5283	K=1.9	10	-0.5265	K=2.5	26	-0.6340**
	√X			-----			-0.5439			-0.5447n.s.			-0.6297
	X <sup>2</sup>			-----			-0.5439*			-0.4949			-0.6192
All	X	K=2.5	156	-0.5862	K=1.0	150	-0.2651	K=2.8	98	-0.2757	K=2.8	90	-0.5383
	√X			-0.6205**			-0.2064			-0.2796**			-0.5642**
	X <sup>2</sup>			-0.5046			-0.3236**			-0.2658			-0.4791
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.6540	K=1.0	33	+0.1903†	K=2.8	10	+0.2956†	K=1.6	24	-0.2310
	√X			-0.7241**			+0.1970†n.s.			+0.2776†			-0.2820n.s.
	X <sup>2</sup>			-0.5380			+0.1791†			+0.3191†n.s.			-0.1465
C	X	K=1.9	84	-0.3980	K=1.0	60	+0.1599†	K=1.0	46	-0.2229	K=1.0	28	-0.6098
	√X			-0.4202**			+0.1802†n.s.			-0.2247n.s.			-0.6509**
	X <sup>2</sup>			-0.3482			+0.1295†			-0.2160			-0.5219
I	X	K=1.6	39	-0.5309	K=1.0	42	-0.2173	K=1.6	32	-0.4515	K=1.0	12	-0.6539*
	√X			-0.5226			-0.1271			-0.4570**			-0.6313
	X <sup>2</sup>			-0.5350**			-0.3405*			-0.4332			-0.6432
O	X	-----	---	-----	K=1.9	15	-0.6073*	K=2.5	10	-0.3148	K=2.5	26	-0.5705**
	√X			-----			-0.6058			-0.3127			-0.5644
	X <sup>2</sup>			-----			-0.6035			-0.3231n.s.			-0.5634
All	X	K=1.6	156	-0.5368	K=1.0	150	-0.2295	K=2.8	98	-0.2156	K=2.8	90	-0.5454
	√X			-0.5589**			-0.1672			-0.2085			-0.5550**
	X <sup>2</sup>			-0.4840			-0.3026**			-0.2263*			-0.5122

\* Significant at 0.05.  
 \*\* Significant at 0.01.  
 n.s. Not significant.

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	K=1.9	33	-0.7854	K=2.2	33	-0.2265	K=1.0	10	-0.8448	K=1.6	24	-0.3523
	$\sqrt{X}$	x=1.1		-0.8066**	x=1.4		-0.2209	x=1.7		-0.8642**	x=0.5		-0.4178*
	X <sup>2</sup>			-0.7174			-0.2392n.s.			-0.8019			-0.2527
C	X	K=2.8	84	-0.6045	K=2.8	60	-0.4323	K=1.6	46	-0.4189**	K=1.0	28	-0.6507
	$\sqrt{X}$	x=1.4		-0.6123**	x=2.0		-0.4664**	x=1.4		-0.4104	x=0.5		-0.6518**
	X <sup>2</sup>			-0.5709			-0.3650			-0.4238			-0.5910
I	X	K=1.6	39	-0.5449	K=1.0	42	-0.2891	K=1.6	32	-0.5035	K=2.2	12	-0.6173
	$\sqrt{X}$	x=0.5		-0.5315	x=1.4		-0.2065	x=0.5		-0.5193**	x=0.5		-0.6068
	X <sup>2</sup>			-0.5612**			-0.3737*			-0.4669			-0.6320*
O	X	-----	---	-----	K=1.9	15	-0.5893	K=2.8	10	-0.3259	K=2.8	26	-0.5705
	$\sqrt{X}$				x=0.5		-0.6057*	x=0.5		-0.3596n.s.	x=0.5		-0.5793**
	X <sup>2</sup>						-0.5493			-0.2648			-0.5422
All	X	K=2.5	156	-0.7308	K=2.5	150	-0.5379	K=2.5	98	-0.4598	K=2.8	90	-0.7142
	$\sqrt{X}$	x=1.7		-0.7550**	x=1.1		-0.5912**	x=1.1		-0.5131**	x=1.1		-0.7941**
	X <sup>2</sup>			-0.6525			-0.4187			-0.3406			-0.5246
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	K=1.0	33	-0.6516	K=1.0	33	+0.1855†	K=1.3	10	-0.7949	K=1.6	24	-0.2190
	$\sqrt{X}$	x=0.5		-0.7203**	x=0.5		+0.1964†n.s.	x=3.0		-0.7892	x=0.5		-0.2721n.s.
	X <sup>2</sup>			-0.5410			+0.1630†			-0.7950**			-0.1365
C	X	K=1.9	84	-0.4209	K=1.9	60	-0.3228	K=1.3	46	-0.3046	K=1.0	28	-0.5913
	$\sqrt{X}$	x=0.5		-0.4330**	x=3.0		-0.3431**	x=2.0		-0.3092*	x=0.5		-0.6228**
	X <sup>2</sup>			-0.3895			-0.2839			-0.2812			-0.5133
I	X	K=1.6	39	-0.5439	K=1.0	42	-0.2764	K=2.2	32	-0.3822	K=1.0	12	-0.6394*
	$\sqrt{X}$	x=0.5		-0.5309	x=1.1		-0.1936	x=0.5		-0.3721	x=0.5		-0.6307
	X <sup>2</sup>			-0.5595**			-0.3742*			-0.3957*			-0.6022
O	X	-----	---	-----	K=1.9	15	-0.3848	K=1.0	10	+0.5650†	K=1.6	26	-0.4629
	$\sqrt{X}$				x=0.5		-0.3933n.s.	x=1.7		+0.4870†	x=0.5		-0.4683*
	X <sup>2</sup>						-0.3600			+0.6353†*			-0.4225
All	X	K=1.6	156	-0.6138	K=1.9	150	-0.4686	K=1.0	98	-0.4019	K=2.8	90	-0.6581
	$\sqrt{X}$	x=1.0		-0.6209**	x=1.4		-0.5146**	x=1.7		-0.4654**	x=0.8		-0.6859**
	X <sup>2</sup>			-0.5876			-0.3605			-0.2630			-0.5511

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

† The sign of the correlation coefficient is reversed from what would be expected from theory.

APPENDIX B, TABLE 23. HIGHEST CORRELATION COEFFICIENTS OBTAINED, USING OPIE'S COMPETITION INDEX  $I_0$

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	BAF=	33	-0.7735	BAF=	33	-0.1785	BAF=	10	-0.2750	BAF=	24	-0.3633
	√X	14.40		-0.7924**	14.40		-0.1815n.s.	69.70		-0.3046n.s.	27.22		-0.4254*
	X <sup>2</sup>	K=2.2		-0.6961	K=2.2		-0.1674	K=1.0		-0.2126	K=1.6		-0.2634
C	X	BAF=	84	-0.4870	BAF=	60	-0.1481	BAF=	46	-0.3375*	BAF=	28	-0.6563
	√X	8.89		-0.5174**	8.89		-0.1710n.s.	41.24		-0.3348	69.70		-0.6694**
	X <sup>2</sup>	K=2.8		-0.4098	K=2.8		-0.1061	K=1.3		-0.3370	K=1.0		-0.5833
I	X	BAF=	39	-0.5413	BAF=	42	-0.2066	BAF=	32	-0.5595	BAF=	12	-0.6290*
	√X	27.22		-0.5333	69.70		-0.1122	27.22		-0.5766**	69.70		-0.6178
	X <sup>2</sup>	K=1.6		-0.5449**	K=1.0		-0.3298*	K=1.6		-0.5169	K=1.0		-0.6057
O	X	-----	---	-----	BAF=	15	-0.5283	BAF=	10	-0.5265	BAF=	26	-0.6340*
	√X			-----	41.24		-0.5102	19.31		-0.5447n.s.	11.15		-0.6297
	X <sup>2</sup>			-----	K=1.3		-0.5439*	K=1.9		-0.4949	K=2.5		-0.6192
All	X	BAF=	156	-0.5842	BAF=	150	-0.2651	BAF=	98	-0.2758	BAF=	90	-0.5383
	√X	11.15		-0.6188**	69.70		-0.2064	8.89		-0.2796**	8.89		-0.5642**
	X <sup>2</sup>	K=2.5		-0.5027	K=1.0		-0.3236**	K=2.8		-0.2658	K=2.8		-0.4791
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	BAF=	33	-0.6540	BAF=	33	+0.1405†	BAF=	10	+0.2955†	BAF=	24	-0.2310
	√X	69.70		-0.7242**	41.24		+0.1594†n.s.	8.89		+0.2775†	27.22		-0.2822n.s.
	X <sup>2</sup>	K=1.0		-0.5380	K=1.3		+0.1062†	K=2.8		+0.3191†n.s.	K=1.6		-0.1465
C	X	BAF=	84	-0.3979	BAF=	60	+0.1598†	BAF=	46	-0.2229	BAF=	28	-0.6098
	√X	14.40		-0.4200**	69.70		+0.1802†n.s.	69.70		-0.2246n.s.	69.70		-0.6509**
	X <sup>2</sup>	K=1.9		-0.3482	K=1.0		+0.1295†	K=1.0		-0.2160	K=1.0		-0.5218
I	X	BAF=	39	-0.5309	BAF=	42	-0.2174	BAF=	32	-0.4515	BAF=	12	-0.6539*
	√X	27.22		-0.5226	69.70		-0.1272	27.22		-0.4570**	69.70		-0.6312
	X <sup>2</sup>	K=1.6		-0.5350**	K=1.0		-0.3405*	K=1.6		-0.4332	K=1.0		-0.6432
O	X	-----	---	-----	BAF=	15	-0.6073*	BAF=	10	-0.3148	BAF=	26	-0.5705n.s.
	√X			-----	19.31		-0.6058	11.15		-0.3127	11.15		-0.5645
	X <sup>2</sup>			-----	K=1.9		-0.6035	K=2.5		-0.3231n.s.	K=2.5		-0.5634
All	X	BAF=	156	-0.5368	BAF=	150	-0.2295	BAF=	98	-0.2156	BAF=	90	-0.5454
	√X	27.22		-0.5588**	69.70		-0.1672	8.89		-0.2085	8.89		-0.5550**
	X <sup>2</sup>	K=1.6		-0.4840	K=1.0		-0.3026**	K=2.8		-0.2263*	K=2.8		-0.5122

\* Significant at 0.05.  
 \*\* Significant at 0.01.  
 n.s. Not significant.

Crown class	Curve form	Slash pine, block 31			Slash pine, block 32			Loblolly pine			Longleaf pine		
		Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r	Best level	No. of obs.	r
<b>P.A.I. in basal area</b>													
D	X	-----	33	-0.8146**	-----	33	-0.2249	-----	10	-0.1962	-----	24	-0.5287
	√X			-0.8091			-0.2176			-0.1789			-0.5492**
	X <sup>2</sup>			-0.7627			-0.2385 n.s.			-0.2263 n.s.			-0.4838
C	X	-----	84	-0.3190	-----	60	+0.0856†	-----	46	-0.3238	-----	28	-0.5493**
	√X			-0.3423**			+0.0895† n.s.			-0.3113			-0.5424
	X <sup>2</sup>			-0.2755			+0.0782†			-0.3459*			-0.5470
I	X	-----	39	-0.5860**	-----	42	-0.2497	-----	32	-0.5525	-----	12	-0.6230
	√X			-0.5851			-0.2197			-0.5449			-0.6278*
	X <sup>2</sup>			-0.5816			-0.3030*			-0.5639**			-0.6012
O	X	-----	---	-----	-----	15	-0.4246	-----	10	-0.4858	-----	26	-0.5391
	√X			-----			-0.4293 n.s.			-0.4849			-0.5243
	X <sup>2</sup>			-----			-0.4090			-0.4903 n.s.			-0.5479**
All	X	-----	156	-0.5292	-----	150	-0.2306	-----	98	-0.1546	-----	90	-0.3209
	√X			-0.5481**			-0.2273			-0.1486			-0.3296**
	X <sup>2</sup>			-0.4745			-0.2348**			-0.1650 n.s.			-0.3016
<b>P.A.I. in d.b.h.o.b.</b>													
D	X	-----	33	-0.7238	-----	33	-0.0573	-----	10	+0.0741†	-----	24	-0.3417
	√X			-0.7529**			-0.0372			+0.0904† n.s.			-0.3552 n.s.
	X <sup>2</sup>			-0.6405			-0.0992 n.s.			+0.0427†			-0.3149
C	X	-----	84	-0.3512	-----	60	+0.2734†	-----	46	-0.1640	-----	28	-0.5931
	√X			-0.3702**			+0.2793†*			-0.1458			-0.6040**
	X <sup>2</sup>			-0.3113			+0.2606†			-0.2002 n.s.			-0.5625
I	X	-----	39	-0.5962**	-----	42	-0.2023	-----	32	-0.4710	-----	12	-0.6073
	√X			-0.5958			-0.1684			-0.4584			-0.6118*
	X <sup>2</sup>			-0.5917			-0.2649 n.s.			-0.4936**			-0.5890
O	X	-----	---	-----	-----	15	-0.5209	-----	10	-0.4263	-----	26	-0.5822**
	√X			-----			-0.5231*			-0.4128			-0.5754
	X <sup>2</sup>			-----			-0.5093			-0.4547 n.s.			-0.5757
All	X	-----	156	-0.5474	-----	150	-0.1749	-----	98	-0.1386	-----	90	-0.4334
	√X			-0.5636**			-0.1602			-0.1291			-0.4422**
	X <sup>2</sup>			-0.5002			-0.2018*			-0.1568 n.s.			-0.4103

\* Significant at 0.05.

\*\* Significant at 0.01.

n.s. Not significant.

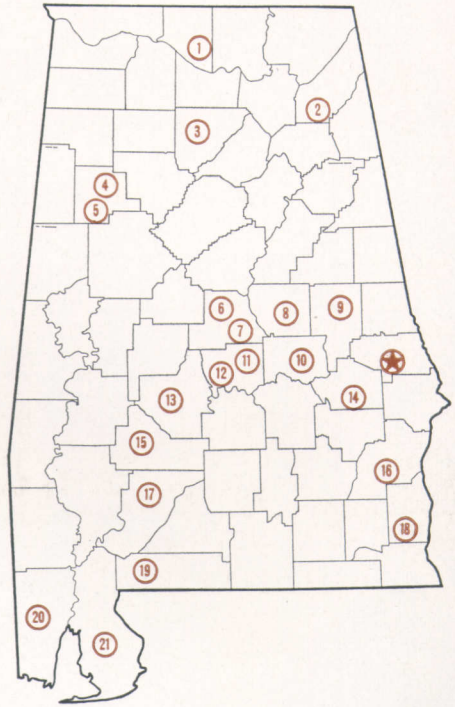
† The sign of the correlation coefficient is reversed from what would be expected from theory.





## AGRICULTURAL EXPERIMENT STATION SYSTEM OF ALABAMA'S LAND-GRANT UNIVERSITY

With an agricultural research unit in every major soil area, Auburn University serves the needs of field crop, live-stock, forestry, and horticultural producers in each region in Alabama. Every citizen of the State has a stake in this research program, since any advantage from new and more economical ways of producing and handling farm products directly benefits the consuming public.



### Research Unit Identification

★ Main Agricultural Experiment Station, Auburn.

1. Tennessee Valley Substation, Belle Mina.
2. Sand Mountain Substation, Crossville.
3. North Alabama Horticulture Substation, Cullmar.
4. Upper Coastal Plain Substation, Winfield.
5. Forestry Unit, Fayette County.
6. Thorsby Foundation Seed Stocks Farm, Thorsby.
7. Chilton Area Horticulture Substation, Clanton.
8. Forestry Unit, Coosa County.
9. Piedmont Substation, Camp Hill.
10. Plant Breeding Unit, Tallassee.
11. Forestry Unit, Autauga County.
12. Prattville Experiment Field, Prattville.
13. Black Belt Substation, Marion Junction.
14. Tuskegee Experiment Field, Tuskegee.
15. Lower Coastal Plain Substation, Camden.
16. Forestry Unit, Barbour County.
17. Monroeville Experiment Field, Monroeville.
18. Wiregrass Substation, Headland.
19. Brewton Experiment Field, Brewton.
20. Ornamental Horticulture Field Station, Spring Hill.
21. Gulf Coast Substation, Fairhope.