Photographic Determination of Pulpwood Volume in Rick-Piled Storage Yards
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Photographic Determination of Pulpwood Volume in Rick-Piled Storage Yards

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The pulp and paper industry is an important component of the economy of several sections of the United States. Among these are the Southeast, the Pacific Northwest, the Lake States, and the upper portion of the Northeast.

Mills that make up this industry are faced with recurring bookkeeping and taxation problems that can only be answered by inventories of wood supply in their storage yards. With some companies this inventory consists merely of deducting the volume of wood consumed by the plant from the volume of the incoming wood, whereas with others a rather precise estimate of the volume is required. These pulpwood inventories are usually most effective if they indicate the situation as of the specific moment that represents the beginning or end of a given fiscal period. This fiscal period is often a month in length, but it may be as long as 3 months. Since existing methods of obtaining such inventories are rather slow and time consuming, the resulting estimates apply to a period, rather than to a moment. Usually the time required to make such an estimate at an average mill is approximately 1 day with a crew of 6 to 10 men. Therefore, some error is encountered because of receipt and utilization of material during the day. For this reason, it is believed that considerable saving could be effected if some economical method could be devised by which the conditions existing at a given moment could be frozen until the inventory was completed. At present the only practical way of doing this is to record the conditions on photographs and then analyze the photographs for the volumetric information.

The existing procedures for estimating pulpwood volume in the woodyard depend on the method used to stack the wood.
Some mills use “jackstraw” heaps (Figure 1-A) where the pulpwood bolts are merely piled in irregular conical piles. Other mills use the so-called “rick” method (Figure 1-B) where the wood is stored in long piles with the bolts more or less parallel. The herein reported study at the Agricultural Experiment Station of the Alabama Polytechnic Institute refers to the inventory of woodyards in which wood is stored in the latter manner.
PHOTOGRAPHIC DETERMINATION of PULPWOOD VOLUME

When pulpwood is stored in ricks, the problem of woodyard inventory is simplified. The procedure is precisely the same as that used to measure stacks of pulpwood in the woods. However, the scale of operation is much larger. The length and average height of the rick are determined and used to compute its cross-sectional area. This total cross-sectional area is then divided by the cross-sectional area of a cord, or other unit, to yield the total volume.

There are several problems, however, that make the inventory somewhat more complex than is indicated. These problems are:

1. *Pile beds*, which must be contour-mapped before the pile is constructed so that the wood volume below the horizontal datum plane can be computed.

2. "*Broken*" *piles*, which result from the removal of the wood (Figure 2). The ricks are usually broken up with a clam-shell crane, which knocks all semblance of regularity out of the pile. Such piles must be contour mapped and the volume computed from the contour information.

3. "*Scatter-wood*" which consists of spillage between the railroad cars and pile and also of material that falls or is knocked from the top of the pile. Since scatter-wood is usually a rather constant item, this problem is not acute. After the volume of scatter-wood has been determined several times, an average volume can be derived and used without disturbing the bookkeeping system.

FIGURE 2. A broken pile or rick of pulpwood.
The cost of a physical inventory is, of course, dependent upon the size and complexity of storage yard and on desired degree of precision. In the case of one mill, whose yard contains three major stacks (average between 800 to 1,000 feet in length and 6 to 8 ricks wide) plus several smaller stacks, the average cost of such an inventory, based on one year’s records, was $135. The maximum cost was $263 and the minimum $58. This does not mean that all of these stacks were measured every time. If a pile had not been changed by the addition or removal of material since the last inventory, the volume computed at that time was accepted and the new work concentrated on piles that had undergone some change.

**REVIEW OF LITERATURE**

Stockpile inventory has claimed the attention of photogrammetrists from time to time, but little information concerning such inventories has appeared in print. One reason is that most inventory work of this nature has been done by professional aerial survey organizations that are understandably reluctant to reveal trade secrets. A few items, however, have appeared. An anonymous note in one of the West Coast trade journals (the specific reference data have been lost) mentions use of aerial photographs to determine the number of logs floating in a sawmill log pond. Apparently no attempt was made to classify the logs into size classes or to estimate the volume of material present. Losee (28) describes how the Canadians are using aerial photographs as reports on current conditions of river drives by counting the number of logs or pulpwood bolts in any given part of the river. Kahre (21) mentions that boom inventories involving bolt counts were made as early as 1919.

Prof. K. D. Jackson (19), University of Toronto, has developed a method of determining the volume of a jackstraw heap of pulpwood by means of a photo-theodolite for the photographs and a stereo-comparator for spot height determinations on 10- or 20-foot centers over the pile. This method closely parallels the conventional transit method. It apparently has had little opportunity for acceptance because of the development of aerial photogrammetric procedures that perhaps are somewhat easier to use. These new procedures were originally described by Warburton (45) and are possibly the result of investigations into the photogrammetric inventory of other stored materials.
The photogrammetric method consists of photographing the heap from the air and then utilizing these photographs in a precise plotting machine, such as the Multiplex, Kelsh Plotter, or Wild A-6 or A-8 Autograph, to plot contours of the heap. The volume under the contoured area can then be rather easily computed. J. H. Cornell (11), Photographic Survey Corporation, Toronto, Canada, states the error should not exceed 10 per cent and probably would be considerably smaller. Young (49), University of Maine, has made an investigation of this procedure, which may lead to its adoption in this country. Young states the cost of such an inventory would be approximately the same as that of the existing ground-transit method if the yard contains at least four or five piles and the photographic airplane does not have to be flown over 100 miles. It is also his opinion that the photogrammetric method is somewhat more precise than the conventional procedure, since the contours are plotted as they lie instead of being interpolated between points of known elevation. This procedure is similar to the coal pile inventories reported by Hallert and Fagerholm (17), Haller (15), and the International Society of Photogrammetry (16). In the latter case, errors have been held to 0.5 to 1.0 per cent of total volume in repeated inventories at Vienna (Austria) Gas Works. While these references apply to various types of stockpile inventory, as can be seen none applies to the problem of rick-piled pulpwood.

METHOD OF STUDY

Because of the expensive nature of photogrammetric equipment and the high cost of aerial photography of small areas, the project had to be conducted as a survey of the various possible solutions to the problem. These solutions were set up and evaluated on the basis of existing information available through library research and from correspondence and discussions with men engaged in various areas of the field of photogrammetry. The application of existing principles, procedures, and equipment to the specific problem of woodpile inventory was carried out in a theoretical manner. Proposed inventory procedures were accepted or rejected on the basis of existing information rather than from the evidence of a series of actual trials. The experimental work was limited to a series of oblique photographs, taken in the laboratory under known conditions of position, azimuth,
altitude, and depression angle, with a non-calibrated press-type camera. These photographs were used in an attempt to evaluate several methods of space resection and orientation and to test several formulas used to determine object dimensions. In addition, the same camera was used to take several series of aerial oblique and vertical photographs of a storage yard. Because of the inadequacy of this equipment, none of the experimental work could be used with any degree of confidence and, as a result, was discarded.

SCOPE of STUDY

In an attempt to examine several possible solutions to the problem, these techniques were considered:

1. The use of terrestrial horizontal photographs taken with a photo-theodolite or camera-transit from fixed towers and analyzed by standard graphical and mathematical methods.

2. The use of aerial oblique photographs also analyzed by standard mathematical methods.

3. The use of large scale conventional vertical aerial photographs analyzed by:
   (a) Simple parallax measuring instruments such as the parallax bar or stereocomparator; or
   (b) Precise plotting equipment, such as the Multiplex, the Kelsh Plotter, and the Wild A-6 or A-8 Autographs.

4. Large scale vertical aerial photographs taken with cameras designed to eliminate image-motion, both of the individual frame and continuous strip types, with the analysis done by the special methods designed for use with such photography.

5. The use of radar profiling equipment.

RESULTS of the INVESTIGATION

TERRESTRIAL HORIZONTAL PHOTOGRAPHY

The underlying idea behind this approach to the problem is to use standard methods of terrestrial photographic analysis to measure the lengths of the ricks, to sub-divide the ricks into segments of known length, and to measure the rick height at these sub-divisions to establish profiles of the ricks similar to those found by the existing manual procedure. The analysis for volume
can then be carried out in the conventional manner. Either the trapezoidal rule, the polar planimeter, or the dot grid can be used for computing the cross-sectional area.

**Instruments and Equipment.** Terrestrial photogrammetry is based on the use of some instrument that provides a photograph, the optical center of which can be found and the position in space and orientation of which can be predetermined. This eliminates ordinary cameras, and makes it necessary to use certain specialized pieces of equipment known as “photo-theodolites” or “camera-transits.” As far as can be determined, no American firm now manufactures either of these instruments; but several of the European instrument manufacturers build photo-theodolites.\(^1\) A typical European built photo-theodolite consists of a 10 × 15-cm. (approximately 3.9 × 5.9 inches) plate camera with 165-mm. (6.5 inches) focal-length lens and a 1-second theodolite. With full equipment, an instrument of this type costs about $3,500, including import duties. Since full equipment would not be needed for storage yard inventory, it is possible that the cost could be considerably reduced by purchase of only the basic instrument and tripod.

The camera-transit is apparently an American development resulting from the need for supplemental ground control in topographic mapping from aerial photographs. King (23, 24) describes how such an instrument was designed and built for the U. S. Forest Service. This instrument consisted of a standard large U. S. Forest Service K & E transit with the standards widened and the telescope replaced by a 5 × 7-inch camera. This camera had a cast aluminum cone and was designed to use a standard 5 × 7-inch Graflex plate holder. The lens was a 12-inch, 4 element Goerz, fitted to an Acme No. 3 shutter. The telescope level vial and vertical circle were attached to the camera. With the instrument properly adjusted, the orientation of the optical axis could be read directly from the horizontal and vertical circles.\(^2\)

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\(^1\) The following firms are known to build photo-theodolites of the type considered in the course of this study: Officine Galileo, Florence, Italy; Wild-Heerbrugg Instruments, Inc., Heerbrugg, Switzerland; and Zeiss-Aerotopograph GMBH, Munich, West Germany.

\(^2\) Fairchild Camera and Instrument Corporation of Syosset, N.Y., manufactured a few similar camera-transits for the U. S. Government, but has since discontinued their production.
It is doubtful that a camera-transit could be obtained at the present time except on a custom basis. Just how much such a custom-built instrument would cost is difficult to estimate but it probably would be comparable in price to the photo-theodolite.

If the two instruments are compared for storage yard inventory, the camera transit apparently is better adapted to the job because of the larger photo-scale produced by its longer focal length (12.0 inches compared with 6.5 inches) and the larger working surface provided by its larger format.

To utilize terrestrial photography efficiently, it is necessary that the photographs be taken from some position of vantage where a good unobstructed view of the ricks can be obtained. In the case of pulpwood stacks consisting of several ricks, the utilization of ground level photographs, as done by Jackson (19) in his jackstraw pile inventory, is not desirable because not enough of the second, third, fourth, or fifth ricks can be seen for profiling. Since the ricks average between 20 and 25 feet in height, the author recommends that the photographs be taken from approximately 50 feet in the air. Thoren, in the coal pile inventory described by Hallert and Fagerholm (17), used a crane to lift his phototheodolite high enough to get a clear view of the pile. This method of obtaining a vantage point has merit because it requires little additional investment in equipment and it gets the camera high enough so that an appreciable area can be seen. Use of the crane, however, ties up an important piece of machinery so that it cannot be used for its normal job of unloading trucks and railroad cars or breaking up piles of wood to move them into the mill. Another problem with the use of the crane is the necessity to resect and orient each photograph, since the precise position and orientation of the camera at the time of exposure would not be known. This lengthens the analysis time, introduces the possibility of considerable accumulated error, and adds to the chance of accidental error. It becomes desirable, therefore, to consider some other approach to the problem.

Probably the most practical means of obtaining a clear view of the yard from elevated points of known position is to utilize platforms mounted on towers. Either the Bilby tower conventionally used for triangulation purposes (3) or a simple platform mounted on a utility pole would be satisfactory.

The Bilby towers are demountable steel towers, built primarily for precise triangulation operations of the U. S. Coast and Geo-
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FIGURE 3. Utility pole tower.
dentic Survey. They would provide excellent instrument stations for the woodpile inventory, but they are expensive.\(^3\)

The utility pole tower (Figure 3) is a suggested alternative to the Bilby tower. A pressure treated 60-foot pole costs about $80 (12). A pile would be more expensive, but it would be sturdier and would reduce the necessity of guying. Such a 60-foot pole or pile should be set about 10 feet into the ground and guyed if necessary. Guying is not desirable, since it tends to clutter the yard and may be dangerous to personnel working in the yard. The completed tower, in place, should not cost much over $150.

**Photographic Procedures.** Considerable thought must be given to placement of towers around the storage yard in order to provide adequate photographic coverage. Two basic problems must be overcome. First, since the positions of the ricks are determined by radial line plotting, every portion of the yard must be clearly visible from at least two towers. Visibility from a third tower is desirable to prevent “weak” intersections, such as occur when the observed point lies on, or near, a line connecting the two instrument stations. Second, to facilitate development of the rick profiles, each rick should be photographed as nearly

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3 In a letter to the author, R. M. Knox, Acting Director of the U.S.C. and G.S., stated the Survey obtains most of its towers from Aermotor Company of Chicago, Ill., at a cost of about $1,000 apiece for 90-foot double towers, purchased in large quantities; 50- or 60-foot single towers, therefore, should cost about $500.
“head on” as possible. A suggested limit of angular viewing of the rick is a 40° angle between the longitudinal axis of the rick and a ray from the camera lens (Figure 4). The towers must be so placed that the entire length of every rick is covered by one or more of these fans.

Photographs are taken from each tower in order to provide continuous photographic cover of the yard from the left extreme to the right extreme. Sufficient overlap between adjacent photographs must be provided so that information can be carried from one photograph to the next. An overlap of 20 per cent is sufficient.

A possible yard coverage pattern is shown in Figure 5. The photographs taken from towers 1, 2, 3, and 4 would be used to plot profiles of the ricks in piles C and D, since each of the photographs taken from the nearer towers, 5, 6, 7, and 8, show so little of the pile length that analysis would be hindered. If, however, a rick on the south side of piles C or D is so low that it is hidden from towers 1 to 4, its profile can be determined from photographs taken from the towers along the southern edge. In such case, it would be necessary to disregard the rule concerning the angle formed by a ray from the camera and the main axis of the pile.

Horizontal and vertical control necessary for mensurational analysis of photographs can be gained through establishment of a network of control points that are flagged or otherwise marked for identification on the photographs (43). Knowledge of the relative positions of the camera stations and the control points is a basic requirement.
Horizontal control can be based on a system of rectangular coordinates by which the position of any point can be established. Such a grid system can be oriented on the cardinal directions or on the main axis of the yard. The course and distance between any two points established on the grid can be found readily without additional ground survey. This facilitates relocation of lost control points as well as determination of rick lengths.

The vertical control should be referenced to a specific datum plane. All height measurements on the ricks will be made from this datum plane, and the volume in the pile below this datum will be computed from contour maps of the pile beds. A convenient datum plane would be the average elevation of the railroad tracks. If a switching "hump" is present there will be some elevation difference, but normally such a hump is not present in a pulpwood storage yard because the yard is not used to assemble trains.

In the case of the camera stations, the elevation of a tack in the floor of the platform should be determined. Then, when the instrument is set-up and adjusted, the actual elevation of the lens can be determined by measuring its height above the tack with a flexible steel tape.

The analysis of the length and height of the ricks of pulpwood is a relatively easy process with this technique if the aforementioned information is available. The use of horizontal terrestrial photographs has found wide application in certain phases of topographic mapping, particularly for supplemental control in areas of high relief. The procedures are standardized.

"Horizontal" photographs, as contrasted to "vertical" or "oblique" photographs, result when the photograph is taken with the optical axis of the camera perpendicular to a plumb line dropped from the camera lens (40). If a properly adjusted phot theodolite or camera-transit is centered over a point and leveled (Figure 6), the vertical line connecting the fiducial marks P₁ and P₂ will be truly perpendicular and will be the "principal line" of the photograph. Similarly, the horizontal line connecting the fiducial marks t₁ and t₂ will be truly horizontal and will mark the "true horizon," since it is the trace of the plane tangent to the earth’s surface through the camera lens. Such horizontal photographs are true records of both vertical and horizontal angles. These angles can easily be determined by graphical or
FIGURE 6. A horizontal photograph showing the true horizon, principal line, and graphical determination of both vertical and horizontal angles. (From Tewinkel.)
analytical methods. As can be seen in Figure 6, the horizontal angle $\beta$ is determined merely by constructing a perpendicular to the true horizon from point $a$ to $a'$ and then measuring the angle $\angle OLa'$, when $L$ marks a point on the principal line that lies at distance $f$ from the principal point. The distance $f$ is equal to the focal length of the camera lens. The angle $\beta$ can also be computed as follows:

$$\beta = \arctan \frac{x}{f}$$

$x =$ distance $Oa' \\
f =$ focal length of lens \\
(x and $f$ in the same units)

In a similar manner the vertical angle $\phi$, from the true horizon to the point of interest, through the point $L$, can be computed:

$$\phi = \arctan \frac{a'a''}{La'}$$

The line $La'$ can be measured or can be computed:

$$La' = f \sec \beta$$

while the line $a a'$ (equal to $a'a''$) must be measured.

Since the pulpwood storage yard is covered by multiple photographs, it is possible to utilize the principle of radial line triangulation to plot the positions of the piles. In this case, the triangulation would be directly analogous to the procedure of secondary triangulation for supplemental control on vertical aerial photographs.

The first requirement for pile location is a base sheet showing the camera stations and control points. The principal lines of the various photographs are plotted on this sheet from the horizontal circle data, as shown in Figure 7. By the procedure described above, the horizontal angles ($\beta$) from the respective principle lines to all points marking the ends of ricks (Figure 7) are computed and plotted on the base sheet. The point where the rays from two camera stations to the same point cross is the true map position of the point. When both ends of a rick have been plotted on the base map, the length of the rick can be measured directly.
Once the rick length has been determined, it becomes necessary to measure the heights of the ricks at definite intervals along the rick length to develop its profile. When such a profile is made by the conventional manual procedure, the interval varies from 100 feet on piles over 1,000 feet in length to 25 feet on piles less than 500 feet long. Since the precision of height measurement on the photographs is somewhat poorer than that possible (with a measuring tape) in the yard itself, it is necessary to reduce the interval and take more height measurements. For that reason, it is probably desirable to reduce the interval to 25 feet on the long piles and to 10 feet on the short piles.

The next step is to divide the rick lengths into the aforementioned intervals and to mark these intervals on the base map (Figure 8). These points are then transferred to the photographs by reversing the point plotting procedure. This is done by measuring the angles from the principal lines to the points on the base map. Since the horizontal photograph is a record of true horizontal and vertical angles, there is no difficulty in this procedure except that errors may be caused by poor measurement techniques. The angles are small, and it is recommended that a vernier type protractor that can be read accurately to one minute of arc be used. These angles are then laid out on the original
FIGURE 8. Dividing ricks into intervals on base map for profile development.

photograph (Figure 9), so that they extend from point L to the true horizon. Perpendiculars are then drawn from the true horizon to the base of the rick. Where these perpendiculars cross the top of the ricks indicates the points at which the rick height will be computed.

The vertical angles from the true horizon to these points along the crest of the rick are then computed by the method described earlier for the determination of angle $\phi$ (Figure 6). The distances from the nadir point (the plumb point directly below the camera lens on the horizontal yard datum plane, or the plotted horizontal position of the camera lens on the map) to the points along the rick are then carefully measured on the map sheet. The differences in elevation between the horizontal datum plane through the camera lens and the points along the rick crest are computed by multiplying the map distances obtained above by the tangents of the corresponding vertical angles. To obtain the heights of the points along the rick crest above the yard datum plane, the vertical distances computed above are subtracted from the elevation of the camera lens above the yard datum plane. This provides the profile needed for the volume estimate. An example of this computation is shown in Appendix I.

From this point the inventory operations are identical with
FIGURE 9. Determination of points along the rick on the photograph where rick heights will be measured.
those used in the conventional manual approach. The cross-sectional area of the rick is obtained by use of the trapezoidal rule, Simpson's rule, a polar planimeter, or a dot grid. The photoelectric planimeter developed by Nash (34) can also be used. The rick volume is then obtained by dividing the cross-sectional area of the rick by the cross-sectional area of the pulpwood volume unit in use.

Up to the present we have only considered the cross-sectional area and volume above the datum plane. It must be remembered that a certain amount of wood lies below this plane. To determine the volume of this wood, it is necessary to have a 1-foot contour map of the pile beds. This can only be made before the pile is constructed and must be done on the ground by conventional topographic surveying methods. Usually such a survey is needed every time a pile is exhausted, since the contours of the bed will be changed by the accumulation of bark and other debris or by cleaning out this debris with a bulldozer. The bed contour map should be made a part of the base map of the yard so that, when the rick ends are located by radial triangulation, they can be plotted directly on the contour map. The profile of each rick below the yard datum plane is found on a transect drawn on the map from one end of the rick to the other. The profile corresponding to this transect is then made from the contour map. The cross-sectional area of this profile and the corresponding wood volume is computed as before. The volume of wood below the datum plane is added to that above the datum plane to yield the total volume.

This method of yard inventory will not solve the problem of the broken pile (Figure 2). It may, however, be possible to plot the ground plan of such a heap by radial line intersection and then to take a series of spot heights to make a contour map of the pile. Volume of the pile could then be determined from the volumes in the strata defined by the contour lines.
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Estimated cost. The costs of equipment and inventory are estimated as follows:

Cost of instrument .......................................................... $2,000.00
Cost of control survey ...................................................... 200.00
Cost of 10 towers @ $150 ................................................... 1,500.00

Total investment ............................................................. $3,700.00

Investment cost per inventory (based on 12 inventories per year for 30 years): $3,700.00 ÷ 360 = 10.28
Labor per inventory, 20 hours @ $3 per hour .................................. 60.00
Survey maintenance ........................................................... 25.00

Miscellaneous expenses per inventory (developing negatives, making prints, drawing paper, etc.) ............................. 5.00

Total cost per inventory ....................................................... $100.28

LARGE SCALE CONVENTIONAL VERTICAL AERIAL PHOTOGRAPHY

Instead of being taken from fixed known ground positions with the camera axis precisely horizontal, the photographs are taken from an aircraft at unknown positions with the camera axis theoretically vertical. Vertical photography presents a plan view of the storage yard, simplifying construction of a planimetric map of the yard and measurement of rick lengths and widths. Stereoscopic analysis of overlapping pairs of vertical photographs, by some form of parallax-measuring instrument, yields rick heights. There are two possible approaches to the measurement of rick dimensions on vertical aerial photographs. The first of these involves use of light camera equipment of the reconnaissance type and simple parallax measuring instruments, such as the parallax bar. Such equipment is admittedly crude and can only provide a low order of precision. However, it is relatively low in cost and its use merits some study. The second approach requires use of such heavy photographic and plotting equipment that it can only be carried out by professional aerial survey concerns. It can yield highly precise inventories but, naturally, the cost of such inventories would be high.

LOW COST APPROACH. The photographic equipment needed for this approach is less expensive than that needed for the terrestrial method. The camera should be one that is designed for aerial operations. It should be light, compact, and easily operated, since even in a slow aircraft there is relatively little time available to take the required photographs. The lens should be mounted at the end of a rigid metal cone rather than at the end of a bellows, to avoid bellows flutter when the camera is exposed to
the slipstream. The shutter should be of the between-the-lens type, since a focal plane shutter creates a distorted photograph. Shutter speed settings should range from 1/100 second to 1/500 second. High shutter speeds facilitate taking low-altitude photographs with a minimum of blur or image-motion. The focal length of the lens should be about 12 inches in order to provide a photographic scale large enough to facilitate analysis. The maximum lens aperture should be at least f/4.5 to permit taking photographs under a variety of light conditions. The camera should be equipped with a film magazine for either roll or sheet film that will recycle in no more than 3 seconds. The focal plane should be the open type, and the film flattened by a vacuum or air pressure system rather than by a glass pressure plate. Accurately located collimation or fiducial marks should be located in the focal plane to facilitate photogrammetric analysis. The picture format (size) should be 5 × 7 inches or larger to provide an adequate working surface.

As far as can be determined, such a camera is not now manufactured for civilian use. Existing civilian cameras, such as the Fairchild F-275, are designed for pictorial rather than photogrammetric operations. They fail to meet specifications necessary for woodyard inventory, with particular regard to shutter, focal plane, and fiducial marks. It may be possible, however, to obtain a suitable military camera from a dealer in surplus equipment. The military camera that probably would be most suitable is the U. S. Navy type F-56 (13). It is perhaps heavier than is ideal, but it has been used successfully in a Cessna 195 plane. It is doubtful that this camera could be used in a truly light airplane because of the space needed for the mount. It is available in four different sizes with lenses of different focal lengths. The most desirable sizes are the 8 1/4-inch f/4.0 and the 20-inch f/5.6. Both the Fairchild and Navy cameras have a 7 × 7-inch format, a shutter speed ranging from 1/75 to 1/225 second, and can be either manually or electrically operated. When electrically driven, they have a recycling speed of one exposure per second. The time interval between exposures can be controlled with an intervalometer. The price of such surplus cameras varies, and it is difficult to state a definite figure. However, it would

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4 Manufactured by Fairchild Camera and Instrument Corporation, Robbins Lane, Syosset, N. Y.

5 See any issue of Photogrammetric Engineering for advertisements of dealers in this type of equipment.
probably be about $1,000, including the mount and intervalometer.

The U. S. Air Force type K-20 and K-25 cameras can also be considered (13). These cameras are considerably smaller than the F-56 and can be used from light aircraft. The two cameras are similar, except that the K-20 is hand-held and hand-operated and the K-25 is mechanically mounted and electrically driven. The lenses of both cameras are 63/8-inch focal length, f/4.5. Their shutters are the between-the-lens type and have a speed range from 1/125 to 1/500 second. Their format size is 4 × 5 inches. The recycling speed of the K-25 is approximately one exposure per second. Instead of an intervalometer, a manual push-button release is used to activate the shutter. Here again it is difficult to state a definite price, but the K-20 should be available for approximately $200 and the K-25 for $350. These are excellent cameras; but, because of their small format size and short focal length, it is doubtful that they would prove satisfactory for woodyard inventory operations.

Other available military cameras that meet the basic specifications are too large and heavy for use in light aircraft.

The second major item of equipment is the airplane. This aircraft should be capable of being flown at a relatively low speed without loss of control. The best solution would probably be the use of light, fabric covered, high wing, training monoplanes such as the Piper J-3 or the Aeronca Champion. Heavier, flap equipped aircraft, such as the Cessna 170, could also be used. If a hand held camera is used, the pulp company need not invest in an airplane. Suitable craft are usually available at local airports for charter for fees ranging from $12 to $25 per hour with pilot. Use of such a camera as the F-56 or K-25, which requires a mount, necessitates extensive cockpit modifications to accommodate the mount. It is unlikely that a charter operator would be willing to so modify his plane. Therefore, the pulp company would be faced with purchase of a suitable aircraft. Though purchase and maintenance of such a ship would not be justified by woodyard inventory alone, it could probably be used for additional purposes.

Helicopters have been considered for the job, since they are capable of low speeds, thus making it possible to take photographs at large scales without blur. Short (37) states helicopters can be used successfully for aerial photography now that many
of the vibrations have been eliminated. The U. S. Army Engineers (44) have also found the helicopter a satisfactory camera platform for taking vertical aerial photographs for the purpose of extending triangulated ground survey control. However, because of the generally high charter rates, use of rotary winged aircraft cannot be considered feasible for the work under consideration.

The special plotting equipment, aside from that found in a drafting room, consists primarily of an instrument for measuring differential parallax on the photographs. Such instruments are called "stereometers" or "parallax bars" and are available in several forms at different prices. 6

The operation of taking vertical aerial photographs for mapping purposes has been developed through long experience. Harman (18) describes these standardized procedures in detail. Undoubtedly the best approach would be to use these standardized procedures with a camera hung in a mount. However, it may be possible to obtain satisfactory results using a hand-held camera, but this is doubtful since it would be difficult to keep tilt within allowable limits. If an attempt is to be made to use a hand-held camera, it must be equipped with level vials and the photographer must be trained to hold the camera level during each exposure cycle. Since external forces acting on the aircraft and its contents can profoundly affect a level bubble, it is essential that the airplane be flown as steadily as possible. In addition, the flying should be done at a time of day when atmospheric turbulence is at a minimum.

Since the precision of height measurements on vertical aerial

---

6 Parallax bar-mirror stereoscope combinations can be purchased from Wild Surveying Instrument Supply Company, Port Washington, N. Y.; Transmares Corporation, New York City (agents for Zeiss Aerotopograph); or Fairchild Camera and Instrument Corporation, Syosset, N. Y. Such combinations cost approximately $300. Binocular attachments, which may be necessary for satisfactory height measurements, are available for all of these stereoscopes.

The following stereometer-type instruments, designed for contouring or form-lining, in which the parallax bar and stereoscope are combined into one unit, can also be used: The Contour Finder, manufactured by Abrams Instrument Corporation, Lansing, Mich.; the Stereotop II, manufactured by Zeiss Aerotopograph; and the Stereocomparagraph, until recently manufactured by Fairchild Camera and Instrument Corporation. Prices of these instruments range upwards from $450, depending on what attachments are included.

A relatively crude parallax bar can be obtained for approximately $35 from Abrams Instrument Corporation for use with the Abrams pocket lens stereoscopes.
PHOTOGRAPHIC DETERMINATION of PULPWOOD VOLUME

photographs increases as the scale increases (38, 39), it is theoretically essential that the scale of the woodyard photography be as large as possible. The maximum scale that can be attained is dependent upon two factors: image motion or blur and recycling speed. Image motion occurs when the camera shutter is not fast enough to stop the movement of the image over the surface of the negative. This causes points to be imaged as lines. The longer the lines the greater the blur. The usually accepted limit of image motion on vertical aerial photographs is 0.2 mm. or 0.008 inch (40). Image motion is usually considered the major obstacle in obtaining satisfactory aerial photographs at large scales. This is true when individual photographs are considered, but, when stereoscopic photographic coverage is required, recycling speed, rather than image motion, often becomes the limiting factor. The maximum scales that can be obtained under a given set of conditions can be computed using standardized procedures (40). It is doubtful that scales larger than 1:1200 can be obtained with standard equipment. Heller of the U. S. Department of Agriculture, Bureau of Entomology and Plant Quarantine (20), while taking vertical aerial photographs at a scale of 1:1200 with an F-56 camera, apparently was unable to get more than approximately 45 per cent end-lap in either of two runs. Both runs also showed excessive image motion.

To keep shutter speeds high, to keep image motion at a minimum, and to provide exposure latitude for days of low sunlight intensity, a fast panchromatic film should be used.⁷

Before any computational work can be begun, the storage yard and its immediate surroundings must be mapped and a fairly dense net of readily visible vertical and horizontal control points established. The specifications for this survey are the same as for that described under “TERRESTRIAL HORIZONTAL PHOTOGRAPHY,” page 8.

The principal points of the photographs are located on the base map by the method of radial line resection, made possible by the dense net of ground control. This procedure is identical to the construction of a transparent templet radial line plot except that each photograph is fitted individually to the ground control. Ex-

⁷ A good film choice would be Aerographic Super XX, manufactured by Eastman Kodak Company, Rochester 4, N. Y.
cessive tilt in a photograph will become evident when it becomes impossible to resect the position of its principal point from more than three control points.

When the photographs have been resected and the positions of the principal points marked on the base map, the positions of the various ricks are located by secondary radial line intersection. In this process, rays are drawn on transparent templets from the principal points of the resected photographs to the points of interest along the ricks. These templets are then assembled on the base map. The intersections of rays from adjacent photographs to common points, such as the corners of a rick, are the true map positions of those points. This process is continued until the entire storage yard has been mapped and every rick has been shown.

On the resulting map of the yard, the ricks are subdivided for height measurements as was done in the case of terrestrial horizontal photographs. Rays are drawn on the map from these points to the principal points of the applicable photographs. These rays are then traced on the resected photographs. The points at which the rays cross the corresponding ricks on the photographic image are where the height measurements will be made.

By standard methods of spot height measurements with a parallax instrument (7, 39, 40, 41, 48), the height of the rick is determined at each of the aforementioned points. These heights, however, should be measured from the standard datum plane rather than the actual ground level. Since tilt and other photographic distortions usually cause warping of this plane in the stereomodel, it will probably be necessary to construct a correction graph for each stereopair. Corrections read from these graphs are applied to the theoretical absolute parallax of the datum plane. These corrected absolute parallax values are then substituted for those that would otherwise be obtained at ground level.

The computed rick heights are used in constructing a profile of the portion of the rick above the datum plane. The profile of the rick below the datum plane is determined from the contour map of the rick bed. From this point the computational procedures are identical to those described under Terrestrial Horizontal Photography.
The estimated cost of an inventory made using the low cost approach is as follows:

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of camera (F-56 with mount and intervalometer)</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>Cost of stereometer</td>
<td>$300.00</td>
</tr>
<tr>
<td>Cost of control survey</td>
<td>$200.00</td>
</tr>
<tr>
<td>Total Investment</td>
<td>$1,500.00</td>
</tr>
</tbody>
</table>

Investment cost per inventory (based on 12 inventories per year for 20 years): $1,500.00 / 240 = $6.25

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft charter cost per inventory (Cessna 170 with pilot and prorating modification costs)</td>
<td>$40.00</td>
</tr>
<tr>
<td>Labor per inventory, 20 hours @ $3 per hour</td>
<td>$60.00</td>
</tr>
<tr>
<td>Survey maintenance</td>
<td>$25.00</td>
</tr>
<tr>
<td>Miscellaneous expenses per inventory</td>
<td>$5.00</td>
</tr>
<tr>
<td>Total cost per inventory</td>
<td>$136.25</td>
</tr>
</tbody>
</table>

THE HIGH COST APPROACH. The commercial photography used in this approach would be done with mapping cameras at a scale of 1:2400. According to representatives of two of the aerial survey companies (42, 47), the cost of photography for an average-sized yard would probably range from $1,200 to $1,500 per survey, unless a plane was in the immediate neighborhood. The cost might be reduced to as little as $500 if an agreement were made between the aerial survey company and the pulp company that the yard be photographed each month, or if several mills within a 100-mile radius required aerial photography at about the same time.

The same simple basic procedure of mapping and profiling is used with the precise plotters as with the parallax bar equipment. The profiles can be developed utilizing spot heights or, if one of the European machines is used, they can be plotted directly by reversing the Z and Y drive shafts to the plotting table. In the case of the latter scheme, the cross-sectional area of the ricks can be measured either by the trapezoidal rule or by the utilization of a polar or electronic planimeter (34). Such an analysis would cost approximately $100 to $150 per stereo-model, exclusive of photographic costs. Information needed for this analysis includes one elevation at each corner of each model area of about 750 feet X 1,250 feet (overlap area between two adjacent photographs) together with a distance approximating the length of the woodpile. This distance should be measured between two points that are readily identifiable on the photographs. According to Woodward (47), the most desirable aerial negative scale would probably be 1:2400 (1 inch = 200 feet) and the plotting scale 1:480 (1 inch = 49 feet).
The estimated cost of an inventory made using the high cost approach is as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of photography (minimum)</td>
<td>$500.00</td>
</tr>
<tr>
<td>Cost of analysis (4 stereomodels @ $125 per model)</td>
<td>$500.00</td>
</tr>
<tr>
<td>Control survey maintenance and amortization cost per inventory</td>
<td>$25.00</td>
</tr>
<tr>
<td><strong>Total cost per inventory</strong></td>
<td><strong>$1,025.00</strong></td>
</tr>
</tbody>
</table>

These two approaches provide what is probably the best solution to the problem of broken piles. This solution is identical to that used in the inventory of jackstraw heaps of pulpwood in which the piles are contour-mapped from stereopairs and the volume of each layer, or stratum, is determined \((11, 15, 16, 17, 45, 49)\). Naturally, the high cost approach would provide the more precise answer because of its superior instrumentation.

**Aerial Oblique Photography**

In this portion of the study, the possibility of utilizing oblique aerial photographs was investigated. In certain respects this method has much in common with the one using horizontal terrestrial photography. However, since the aerial photographs are taken from unknown positions at unknown orientations, much more work is involved in their use. These unknown items, position and orientation, are formally broken down into six elements: the x and y coordinates of the camera station, referred to the yard coordinate system; the elevation of the camera station above the datum plane (the z coordinate); the azimuth of the principal line, referred to the yard coordinate system; the angle of inclination of the camera axis from the vertical (tilt); and the swing, the angle between the principal line and the + y fiducial axis of the photograph. Graphical, instrumental, and analytical approaches to the determination of these elements are available in the literature \((6, 7, 26, 33, 40, 41, 46)\).

The graphical methods, such as the ones devised by D. R. Crone of the Survey of India \((41)\) and R. M. Wilson of the U. S. Geological Survey \((46)\), are subject to considerable amounts of cumulative error. Since the pattern of error accumulation is not constant, it is impossible to obtain a unique solution to the problem of space resection and orientation. This looseness of the graphical methods results in a questionably defined position of the nadir point, which, in turn, has a strong and adverse effect on the precision of subsequent object measurements.
A better solution is to use a mechanical device to determine the position and orientation of the camera and to measure the horizontal and vertical angles needed for the woodyard inventory. The Wilson photoalidade, the standard instrument used by the U. S. Geological Survey for topographic mapping from the oblique wing photographs of Tri-Metrogon photography, is such an instrument (25, 46). It is, however, very expensive and can only be obtained on a custom basis. A new, relatively low priced instrument has been recently announced. It is called the "Photoblique Plotter." Advertising claims state that it can be used "for the determination of horizontal and vertical distances, areas, and relative bearings from a single-high-oblique-photograph," (35). As yet, no other evaluation of this instrument has appeared in print.

The third possible approach to the problem of using aerial oblique photographs in woodyard inventory is to use the methods of analytic geometry. These methods have been thoroughly explored and described by Church (5, 6, 40). They provide for a rigid mathematical solution of space resection and orientation when three control points are available, and they also provide the basis for a least squares solution if more than three control points are used. This mathematical approach has not been used for production photogrammetric operations in the past because of lengthy, tedious computations. The cost of such an operation, if carried out on a desk calculator, would be prohibitive. The advent of electronic computing equipment, however, has renewed interest in the practical applications of this procedure. In some cases the pulp and paper concerns interested in a photogrammetric inventory of their wood storage yards might be equipped with or have access to such computing equipment and could do the work at relatively low cost. Where such equipment is not available or cannot be adapted to the problem of woodyard inventory, it would be necessary to wait until regional computation centers were established where such tedious work could be done cheaply and quickly. Such centers could keep program-

---

8 In a letter to the author, Gerald Fitzgerald, Chief Topographic Engineer of the U. S. Geological Survey, reports that the Wilson Photoalidades used by his organization are made in the Survey's own shops. He also reports that a photoalidade has been built for private use by Photogrammetry, Inc., 7961 Eastern Avenue, Silver Spring, Md. This instrument was priced at approximately $7,000.

9 Manufactured and sold by Photogrammetry, Inc., 7961 Eastern Avenue, Silver Spring, Md., for approximately $385.
ming tapes on file for such standard operations, thus keeping the cost at a reasonable figure.

Because of the lack of precision of the graphical methods, the difficulty in supplying the Photoblique Plotter with suitable tilt data, the lack of performance data on the Photoblique Plotter, and the high cost of the Wilson Photoalidade, this section will deal exclusively with the mathematical approach.

**INSTRUMENTS AND EQUIPMENT.** Best results for an inventory of this type would undoubtedly be obtained from the use of photographs taken with highly precise mapping equipment. Such equipment, however, in addition to being extremely expensive, is not well adapted to taking oblique photographs. If available, it should be used for vertical photography. The method of storage yard inventory using aerial oblique photographs should only be considered when costs must be kept at a minimum and proper computing equipment is available. If such is the case, one can consider the use of light photographic equipment in light aircraft.

The camera should fit the basic specifications listed in the section describing use of conventional vertical aerial photographs, page 21. In addition, it should be capable of hand held operation, even though electrically driven. All cameras discussed in the preceding section can be used. In addition it may be possible to use the Abrams “Baby Explorer,” a 4 × 5-inch hand-held oblique camera\(^\text{10}\) or the U. S. Navy F-56 with a 20-inch f/5.6 lens.

As in the case of the cameras, the aircraft discussed in a preceding section can also be used in this approach. However, since no mount is needed, the aircraft can be lighter; no difficulty should be encountered in chartering a suitable craft at the local airport because no cockpit modifications would be needed.

The mathematical methods of space resection and orientation require a third item of equipment. This is a device for accurately measuring coordinates on the photographs. Such instruments are called “comparators” and are expensive.\(^\text{11}\) A standard comparator is designed to measure coordinates to the nearest 0.0001 inch. Such precision is probably not needed for woodpile inventory operations. It will be assumed that some less expensive method of measurement can be used. Repeated measurements with a

\(^{10}\) Sold by Abrams Instrument Corporation, Lansing, Mich.

\(^{11}\) The comparators built by David W. Mann, Lincoln, Mass., cost approximately $10,000.
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scale graduated in 0.01-inch units might prove satisfactory. However, since tests have not been made, a definite conclusion cannot be stated at this time.

PHOTOGRAPHIC PROCEDURES. The photographic mission should be thoroughly planned before the aircraft leaves the ground. This planning operation must include consideration of several items and balancing of one against the other to obtain a satisfactory set of photographs. Among these items are the following:

1. Photographic scale should be as large as possible so that reliable measurements can be made on the photographs.

2. The airplane should be flown parallel to the longitudinal axis of the main ricks and far enough to the side of the yard so that the camera axis is depressed at approximately a 45° angle from the horizontal. The flight line (the line of the photograph nadir points) should be drawn on the base map before the flight is made and followed as closely as possible by the pilot. Thus, if the above requirements are used and if the airplane is to fly at 500 feet above the woodyard datum plane, the flight line should be located 500 feet away from, and parallel to, the nearest ricks.

3. As in the case of the method utilizing horizontal terrestrial photographs, as much as possible of a “head on” view of the ricks should be obtained. Again the recommendation is made that the angle formed by the intersection of the longitudinal axis of the rick and a ray from the camera station should not be less than 40° (Figure 5). This is complicated by the fact that long focal length, narrow angle lenses are required to provide photographic images that are large enough for reliable measurements. This type of coverage, at the low altitudes and short distances involved, requires a camera that can recycle rapidly. For example, if a U. S. Navy type F-56 camera, with a 7 × 7-inch format and a 20-inch lens, was used in an aircraft flying at 55 m.p.h. at 500 feet above the woodyard datum plane and the camera axis was depressed 45°, the photographs would have to be taken at intervals of approximately 1.2 seconds. This is barely within the recycling ability of the F-56 camera, which has an unusually high recycling speed (13). Average cameras usually require 3 or more seconds to complete a cycle. This may make it necessary to fly higher and farther away, obtaining photographs at a smaller scale.
(4) Adjacent photographs should overlap each other enough to permit carrying information from one photograph to the next and preventing gaps in the cover. It is not necessary, however, to obtain stereoscopic coverage with this method of woodyard inventory.

(5) Each photograph should contain at least three control points for which horizontal and vertical ground coordinates are available. The yard survey should provide enough points of known position to meet this requirement.

(6) Since dense shadows can be detrimental to analysis, photographs should be taken at or near noon so that shadows will be at a minimum. Unfortunately, at noon, the air has developed considerable turbulence that can be troublesome to the pilot and photographer at low altitudes. A minimum altitude of 400 to 500 feet is recommended to reduce “bumping” resulting from turbulence.

(7) As in the case of conventional vertical aerial photography, fast panchromatic film will produce the best results, since relatively fast shutter speeds can be used even under adverse light conditions.

**Computational procedures.** The general problem of space resection and orientation of an aerial photograph has been rigorously solved by Earl Church, Syracuse University, using the methods of analytical geometry \(5, 6, 40\). Since these procedures are well known, no attempt is made here to describe them; instead the reader is referred to the literature.

Before the photographs are taken, the wood storage yard should be surveyed and a topographic base map prepared, as was done with both previously discussed methods of yard inventory. A relatively dense net of accurately located control points should be distributed over the yard so that at least three well spaced control points appear on each photograph.

After the six elements of camera position and orientation have been determined for each photograph, the inventory phase can begin. The first step is to locate the ends of the ricks on the photographs and to measure their photographic coordinates, referenced to the fiducial axes. The corresponding horizontal ground survey coordinates can then be computed using the analytical methods developed by Church \(5, 6, 40\). The rick ends can then be located on the base map and a yard layout map prepared.

The ricks should be subdivided on the map into uniform seg-
PHOTOGRAPHIC DETERMINATION of PULPWOOD VOLUME

FIGURE 10. Determination of horizontal and vertical angles on an oblique photograph.

ments for profiling, as was done in the two preceding sections. The profiling operation can be carried out in either of two ways, using vertical angles, as was done in the case of terrestrial horizontal photography, or by using "heighting" formulas. The first of these two procedures is based on Wilson’s method of determining vertical angles (46). In this method the vertical angle $\phi$ (Figure 10), from the plane of the true horizon to the top of the rick, is computed for each rick interval. The horizontal distance from the nadir to the point is then measured on the map or computed from the coordinate values. The vertical difference between the plane of the true horizon and the top of the rick is determined using the following formula:

$$\overline{AA''} = \overline{LA''} \tan \phi$$

Where: $\overline{AA''}$ = vertical difference between the plane of the true horizon and the top of the rick.

$\overline{LA''}$ = scaled or computed distance from the nadir point to the point on the rick ($\overline{LA''}$ and $\overline{AA''}$ are in the same units).

$\phi$ = vertical angle from the plane of the true horizon to the point on the rick.
To find the depression angle $\phi$ the following formula is used:

$$
\phi = \arctan \frac{y \cos \theta}{x \csc \beta}
$$

Where:

- $y$ = y coordinate of the point on the photograph using the true horizon and the principal line as the coordinate axes.
- $x$ = x coordinate of the point on the photograph.
- $\theta$ = depression angle.

As in the case of the terrestrial method, this develops a reversed profile. To determine the rick heights above the yard datum plane, each value computed above must be subtracted from the elevation of the corresponding camera station. This method is probably superior to that using the "heighting" formulas, since it is not necessary to see the bases of the ricks.

The alternate method of determining rick heights is to use one of the several "heighting" formulas that have been developed. These formulas usually require measurements of the photographic image of the object whose height is being determined. This means the point on the base as well as the point on the top of the rick must be visible or the position (coordinates) of the base point on the photograph must be known. In many cases bases of ricks will be hidden behind adjacent ricks and it will be necessary to compute the photographic coordinates of the points along the bases before the heighting formulas can be applied. To do this the horizontal ground coordinates of the points along the bases of the ricks are measured on the base map. The corresponding photographic coordinates can then be computed by reversing the coordinate computation procedure ($5, 6$). These points should then be located on the photographs by measurement from the fiducial axes. Theoretically these should fall at equal intervals along the base of the rick. Any deviations would be the result of poor mensurational technique.

Rick heights should be computed at each of the points located above. There are many formulas available in the literature for computing these heights. Among these are the following:

1. From Trorey (41), with nomenclature changed to agree with Figure 11.

$$
h = \frac{H(m-k) \cos \eta \cos \mu}{f \cos (\theta + \eta) \sin (\theta + \mu)}
$$
Where: $h =$ height of ground object.

$H =$ elevation of camera station above the yard datum plane.

$m =$ y coordinate distance on photograph from principal parallel to base of object.

$k =$ y coordinate distance on photograph from principal parallel to top of object.

$\eta =$ angle in principal plane from principal line to top of object.

$\mu =$ angle in principle plane from principal line to bottom of object.

$\theta =$ depression angle ($90^\circ$—tilt angle).

$f =$ focal length of camera.

$h$ and $H$ are in the same units. $m$, $k$, and $f$ are in the same units. $m$ and $k$ are positive (+) if they extend downward from the principal parallel and negative if they extend upward. The corresponding angles-off-the-principal line, $\mu$ and $\eta$ also bear the same signs.

**FIGURE 11.** Section view of an oblique photograph projected onto the principal plane.
(2) From Lane (25), with nomenclature changed to agree with Figure 11. The original formula provided the photographic z (vertical) scale rather than the height of the object. For this reason the equation is also so modified that heights can be obtained directly.

\[ h = \frac{(m-k) (f \cot \theta - k) (f \tan \theta + m) \sin \theta \cos \theta}{fH} \]

The symbols are the same as above.
\( f \cot \dot{\theta} \) and \( f \tan \theta \) are constants for the photograph.

The author has also developed a similar formula (see Appendix II):

\[ h = \frac{H \sin (\mu - \eta)}{\cos (t - \mu) \sin (180^\circ - t + \eta)} \]

Where: \( t \) = angle of tilt
The other symbols are the same as above.

After the rick heights have been measured, the rick profile is developed in the usual manner and the volume determined, using the methods described in the section on the use of horizontal terrestrial photography, page 8.

This method, like that using horizontal terrestrial photographs, does not provide a strong solution for the problem of the “broken pile.” A contour map of the heap can be prepared from a net of control points scattered over the pile. The positions and elevations of these points can be determined by the procedures described above. The volume of each stratum is determined from its area and depth.

**Estimated Cost.** The investment and inventory costs are estimated as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of camera (F-56 with intervalometer)</td>
<td>$750.00</td>
</tr>
<tr>
<td>Cost of control survey</td>
<td>$200.00</td>
</tr>
<tr>
<td><strong>Total Investment</strong></td>
<td><strong>$950.00</strong></td>
</tr>
<tr>
<td>Investment cost per inventory (based on 12 inventories per year for 20 years):</td>
<td>3.96</td>
</tr>
<tr>
<td>Aircraft charter costs per inventory (Cessna 170 with pilot)</td>
<td>25.00</td>
</tr>
<tr>
<td>Labor per inventory, 10 hours @ $3 per hour</td>
<td>30.00</td>
</tr>
<tr>
<td>Survey maintenance</td>
<td>25.00</td>
</tr>
<tr>
<td>Computer charges (no information available)</td>
<td>50.00</td>
</tr>
<tr>
<td><strong>Total cost per inventory</strong></td>
<td><strong>$133.96</strong></td>
</tr>
</tbody>
</table>
Large Scale Image-Motion-Compensated Vertical Aerial Photography

The use of aerial cameras designed to reduce the effect of image-motion on the photographs at large scales has been considered because of the possibility of obtaining scales large enough that reliable photogrammetric measurements could be made with simple equipment, thus reducing the cost of inventory.

Remarkable progress has been made in the last decade in developing cameras and camera film magazines that permit taking large scale photographs. Most of these developments have been made to meet military needs for large scale reconnaissance photography, and as yet little of this equipment is available for civilian use. Because of this, the following discussion will merely cover the developments in the field and their possible application to the problem of woodyard inventory without mentioning specific techniques. Enough information has been gathered to make some critical evaluations of the equipment for pulpwood inventory.

There are two basic problems that must be overcome before large scale photography can be obtained. The first is "image-motion," or blur, which occurs when the shutter speed is so slow that points on the ground are imaged on the photographs as lines rather than points. The usual allowable limit of image-movement is 0.2 mm. or 0.008 inch (40). Beyond this point the blur becomes large enough to destroy detail. The second problem is that of "recycling time." This is the time needed to take a picture, clear the focal plane of the exposed negative, and replace it with unexposed film, re-arm the shutter, and be ready to take the next picture. These problems have been discussed in some detail in the preceding sections on vertical and oblique aerial photography.

Two radically different approaches to the problem of large scales have been tried. One approach is relatively conventional, involving cameras whose only modification is a magazine that moves the film along the flight-line at the time of exposure so that there is little or no relative motion between the image of the ground and the surface of the film. Further refinements of this idea have led to development of rapid recycling systems and small photograph formats to get large scale reconnaissance photographs from low-flying jet aircraft. The U.S. Navy CAX-12
70-mm. camera is of this type (2, 10). It can take sharp low altitude photographs, 2¼ inches square, while the plane is flying at near sonic speeds. Its recycling rate is approximately 30 times that of most conventional mapping cameras. It is not known if the U. S. Air Force has developed a camera with such a high recycling rate. It has several cameras and film magazines, however, that are designed for image-motion-compensation (2).

The only privately developed camera utilizing the principle of image-motion-compensation was built by Photographic Survey Corporation of Toronto, Canada (11, 28). This camera uses a swinging mount rather than a moving film magazine to stop image movement. It is apparently designed for normal altitude operations and uses a 24-inch lens to obtain the desired large scale. Since by international agreement Canadian aerial survey organizations cannot operate in the United States, the Canadian camera is not available to Americans at the present time. Its use may be licensed to some American firm in the future, but no hint of that development has yet been received. The author has seen several fine stereoscopic photographs taken with this camera that were at a scale of 1:1000; they had been taken from a plane traveling at 140 miles per hour. Under these conditions the recycling rate was 1.4 seconds per exposure. If these photographs are typical, the camera undoubtedly will be used considerably in applications requiring large scales.

The second approach to the problem of image-motion-compensation is radically different in concept. In this case a shutterless camera is used in which a continuous strip of film is moved by a slit in the focal plane of the camera (8, 10, 32). This is related in principle to the focal plane shutter. To stop relative motion between the image and the film, the film motion is synchronized with the ground speed by an electronic synchronizer. With this device it is possible to get sharp photographs at altitudes as low as 200 feet and speeds up to 1,500 miles per hour. The photographs taken with this camera, commonly called the Sonné camera, are in the form of long, continuous strips. Flat or stereoscopic pictures can be taken. Flat photographs require only one lens pointing straight down, while stereoscopic photographs are taken with two lenses, one vertical and the other inclined about 5° forward or backward along the line of flight. The stereoscopic photographs appear on the same strip, parallel to one another
and slightly offset longitudinally. A special viewer is needed to view these strips stereoscopically.\textsuperscript{12}

Photogrammetrically speaking, the individual frame image-motion-compensation cameras far outperform the Sonné continuous strip camera. The reason is that the individual frame cameras (hereafter referred to as “IMC” cameras) have shutters and freeze the perspective view as of a given fraction of a second. This makes it possible to identify certain points needed for photogrammetric analysis, such as the principal point, nadir, and isocenter, and also their conjugates on the adjacent photos. This, as G. T. McNeil says (30), “enables one to establish a definite film plane for subsequent metrics.” In contrast, the continuous strip camera (hereafter referred to as the “CS” camera) has none of these points, and the only relation one section of the strip has with an adjacent section is that of continuity. There are no geometric relationships that can be used to tie the images on the strip to their correct horizontal positions. Since the “shutter” is wide open all the time, the full effect of lateral motion of the plane (roll or yaw) is shown on the strip. Goddard (14) shows an example of such a distorted photograph. The photograph looks like a tablecloth that had been pushed alternately toward the center from opposite sides of the table. Straight lines appear as continuous S curves. In addition, the CS synchronizer operates from the strongest source of reflected light along the ground. At low altitudes there can be considerable angular motion of objects lying above or below the reference source of light picked up by the synchronizer, thus causing blur (32). If the synchronization device is out of adjustment there can be considerable “shrink” or “stretch” of the photo scale along the line of flight. Up to the present, no method has been devised for correcting this type of distortion.

In spite of the objections to CS photography mentioned in the preceding paragraph, reliable spot heights can apparently be measured on it. Mignery (32) mentions three methods of height measurement: (1) shadows; (2) parallax; and (3) displacement. In his study he found that shadows gave poor results, with parallax and displacement methods being somewhat better. His results in the parallax method were probably not as good as could have been expected if he had had the correct equipment for the

\textsuperscript{12} The camera has been released for civilian use to one company, Chicago Aerial Survey Company of Chicago, Ill.
job. He was forced to improvise a modification of the Harvard parallax wedge when he should have used the stereometer designed for such photography by Chicago Aerial Survey Company. Katz, in an article in Photogrammetric Engineering (22), discusses the measurement of spot heights on CS photography and concludes that satisfactory heights can be determined.

In the opinion of the photogrammetrists (9, 10, 22, 30), the only advantages of the CS photography over the IMC is that it is available for civilian use and that the continuous strip is easier to view than many individual prints.

**Radar Profiling Devices**

Electronic altitude measuring equipment is being used with considerable success for supplemental vertical control for topographic mapping of unexplored areas using Multiplex equipment. Because of this the possibility of utilizing such equipment for profiling ricks of pulpwood was investigated. If the procedure had proved feasible, it would have been possible to fly over each rick and get its profile immediately, thus eliminating several of the steps in any of the photographic methods.

Electronic profiling devices have evolved from electronic altimeters for aircraft. These altimeters are basically of two types (36). The first is a “radio” altimeter based on frequency modulation principles and used as a low altitude clearance indicator. The second uses the principles of radar and is designed primarily for high altitude flying. These unmodified altimeters are not satisfactory for vertical map control and a topographic form has been developed. This instrument was built under the auspices of the National Research Council of Canada, and further development has been carried by several other organizations.

The “radar altimeters” use a constant atmospheric pressure altitude as the datum from which to make measurements to the ground. It must be assumed that this reference is a plane. It may slope, and the slope must be determined from a series of test observations. The profiles obtained with the altimeter can then be adjusted. This requirement for a constant pressure altitude rules out low altitude operations because of atmospheric instability. Normal operating altitudes range from 2,500 to 8,500 feet above ground.

The altimeter is equipped with a 24-inch, parabolic antenna.
PHOTOGRAPHIC DETERMINATION of PULPWOOD VOLUME

A 1.25-cm. wave length is used, and the beam width is 1.5°. This results in a beam diameter at ground level of 28 feet per 1,000 feet of altitude. Reynolds (36) and Blachut (4) state that vertical measurements of a precision within ± 10 feet of true value can be expected. However, a necessary accessory, the “Aircraft Altitude Corrector,” is only reliable to within ± 150 feet.

Obviously, radar profiling devices operated from aircraft are unsuitable for woodyard inventory and need not be further considered.

SUMMARY

Without a doubt, the most desirable of the procedures under consideration is the one utilizing terrestrial horizontal photographs. While it has the disadvantage of a rather high initial investment in equipment, it still has a low per-inventory cost and should produce excellent technical results. The high initial cost of the instrument, towers, and survey can be amortized over a relatively long period. With reasonable care the instrument should last at least 30 years, as should the towers if made of pressure-treated poles or piling. The survey would have to be maintained and changed from time to time, but such surveying operations should not be excessively expensive. Offsetting the high initial costs are the advantages of a relatively low labor cost per inventory; a permanent, checkable record of the conditions existing in the yard as of a given moment; and availability of the record, being able to make the record at the moment desired by the bookkeeping department regardless of all but the most adverse weather conditions. Computations for this procedure are relatively simple. There are no involved orientation or resection problems; and, because of the simplicity of the measurements and computations, there are relatively few opportunities for errors to creep into the work. Because of these factors, it is believed that the method shows considerable promise.

The procedure utilizing commercial, conventional vertical photography, and the precise plotters will undoubtedly provide a better inventory than any other, equalling and probably exceeding the precision of the ground method. It is, however, not economically feasible since the cost of a single inventory ranges from $600 to $1,600, as compared to a manual inventory cost of
$135. There is some question as to the necessity for such heavy charges, and it is possible that they can be reduced to some extent. However, it is doubtful that the method will become competitive with the existing ground procedure. A further disadvantage is that the photographs must be taken under favorable conditions, which means weather with no rain or low-lying clouds. Such conditions are uncontrollable, and it is likely that the aerial survey organization would have difficulty in consistently meeting the timing requirements of the bookkeeping department.

The low cost approach using vertical aerial photographs taken with a mounted camera is only slightly more expensive than that utilizing terrestrial photographs but has several disadvantages that are difficult to overcome. As in the case of the high cost approach, the timing of the photography may not be ideal because of weather conditions. Unless a fully stabilized mount is used, camera tilts may become excessive on days when atmospheric turbulence is severe. Such mounts are expensive and they were not considered in the cost estimates made for this approach.

Tilted photographs cannot be used for radial line mapping, since they are not true records of horizontal angles about their principal points. In addition, height measurements made on such photographs should be viewed with considerable suspicion. Even with truly vertical photographs, parallax measurements made with a parallax bar are subjective in nature and can be erratic. When this is combined with the effects of tilt, the precision of height measurements can be low. Whether or not the use of the parallax bar would be satisfactory in woodyard inventory can only be determined by field trials. Since extensive cockpit modifications are needed for installation of a camera mount, it is doubtful that a chartered aircraft could be used. It is much more likely that an airplane would have to be purchased by the pulp company. Such a purchase could only be justified if the company could find other uses for the airplane that would help absorb the cost. In short, this method shows some promise, but its several drawbacks are rather critical. Much thought should be given to its use before it is accepted.

The low cost approach using an unmounted camera is discredited at the start by the fact that the camera is hand-held. In such a case the photographs are subject to large and erratic tilts making
them unsatisfactory for inventory operations. In addition, the light cameras that would normally be considered for this work are of the reconnaissance type and are not designed or built for critical mensurational operations. The distortion characteristics of their lenses are seldom known. Furthermore, since the camera can be disassembled into several component parts, the interior orientation is not stable. These reconnaissance cameras may also have formats that are too small, lenses with focal lengths that are too short or too long, and shutters of the focal plane type. The probable success of this method of woodyard inventory quite obviously would be limited by even more formidable technical problems than those mentioned in the paragraph on the use of mounted cameras for taking vertical photographs.

The use of aerial oblique photographs might be a very satisfactory way of making a pulpwood inventory if suitable computing equipment were available so that the analytical methods of space resection and orientation could be used. This equipment, unfortunately, is not generally available. This approach to the inventory problems will have to be discarded until regional computing centers are established for carrying out such long, involved, and tedious computations. In addition, this method requires use of fairly precise photographic and mensurational equipment. Ordinary reconnaissance cameras would not be satisfactory. Some relatively low-cost method for accurately measuring coordinates on the photograph would have to be devised. As in the case of all methods involving an airborne camera, weather conditions would strongly influence the timing of the photography. The advantages of oblique over vertical photography is that no aircraft modifications are needed and tilt is not a limiting factor.

Because of its lack of geometric stability, Sonné continuous strip photography cannot be recommended for the woodyard inventory. The Chicago Aerial Survey Company, developer of the Sonné camera, concurs with this opinion (27). If available, the individual frame image-motion-compensated photography could be used; but it is doubtful that any advantage would accrue from its use. The high cost of the photography would offset any advantages over the use of hand-held conventional equipment.

The radar profiling equipment is simply too crude at this stage to be useful for storage yard inventory.
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## APPENDIX I

### COMPUTATION OF RICK VOLUME USING HORIZONTAL TERRESTRIAL PHOTOGRAPHS

(Based on Figure 12)

Computation form for rick profiles:

- Camera Station: 1
- Pile No.: A
- Photo No.: 3
- Rick No.: A-1
- Elevation of camera lens: 50.0'

<table>
<thead>
<tr>
<th>Point No. (from left end of rick)</th>
<th>Angle from true horizon</th>
<th>Tangent of angle</th>
<th>Distance from nadir to point (from map)</th>
<th>Distance from datum thru lens to rick (c×d=)</th>
<th>Rick height (50.0—e =f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yₐ</td>
<td>2°00'</td>
<td>0.03492</td>
<td>1345</td>
<td>47.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°28'</td>
<td>0.02560</td>
<td>1310</td>
<td>33.5</td>
<td>16.5</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°17'</td>
<td>0.02240</td>
<td>1285</td>
<td>28.8</td>
<td>21.2</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°04'</td>
<td>0.01862</td>
<td>1265</td>
<td>23.6</td>
<td>26.4</td>
</tr>
<tr>
<td>Yₐ</td>
<td>0°50'</td>
<td>0.01455</td>
<td>1250</td>
<td>18.2</td>
<td>31.8</td>
</tr>
<tr>
<td>Yₐ</td>
<td>0°53'</td>
<td>0.01542</td>
<td>1215</td>
<td>18.7</td>
<td>31.3</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°01'</td>
<td>0.01775</td>
<td>1225</td>
<td>21.7</td>
<td>28.3</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°07'</td>
<td>0.01949</td>
<td>1205</td>
<td>23.5</td>
<td>26.5</td>
</tr>
<tr>
<td>Yₐ</td>
<td>0°57'</td>
<td>0.01658</td>
<td>1210</td>
<td>20.1</td>
<td>29.9</td>
</tr>
<tr>
<td>Yₐ</td>
<td>0°50'</td>
<td>0.01658</td>
<td>1240</td>
<td>20.6</td>
<td>29.4</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°07'</td>
<td>0.01949</td>
<td>1245</td>
<td>24.3</td>
<td>25.7</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°30'</td>
<td>0.02619</td>
<td>1265</td>
<td>33.1</td>
<td>16.9</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°48'</td>
<td>0.03143</td>
<td>1305</td>
<td>41.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Yₐ</td>
<td>1°58'</td>
<td>0.03434</td>
<td>1330</td>
<td>45.6</td>
<td>4.4</td>
</tr>
</tbody>
</table>

**FIGURE 12.** Cross-sectional profile of a rick of pulpwood.
The cross-sectional area of the rick is computed using the "trapezoidal rule":

Area of main part of stack:
\[ A = \left( \frac{1}{2}y_0 + y_1 + y_2 + \ldots + y_{n-1} + \frac{1}{2}y_n \right)h \]
\[ \frac{1}{2}y_0 = \frac{1}{2}(3.0) = 1.5 \]
\[ y_1 = 16.5 \]
\[ y_2 = 21.1 \]
\[ y_3 = 26.4 \]
\[ y_4 = 31.8 \]
\[ y_5 = 31.3 \]
\[ y_6 = 28.3 \]
\[ y_7 = 26.5 \]
\[ y_8 = 29.9 \]
\[ y_9 = 29.4 \]
\[ y_{10} = 25.7 \]
\[ y_{11} = 16.9 \]
\[ \frac{1}{2}y_{12} = \frac{1}{2}(9.0) = +4.5 \]
\[
\frac{289.8}{25} = 289.8 \\
7247.5 \text{ square feet}
\]

Area of odd-sized segment:
\[ A = \frac{y_{12} + y_{13}}{2} h \]
\[ = \frac{9.0 + 4.4}{2} 15 \]
\[ = (6.7)15 = 100.5 \text{ square feet} \]

Total area of rick:
\[ 7247.5 + 100.5 = 7348.0 \text{ square feet} \]

Using a normal 128-cubic-foot cord as the unit of volume with 6-foot bolts the cross-sectional area of one unit is:
\[ \frac{128}{6} = 21.33 \text{ square feet} \]

The volume of the rick above the horizontal datum plane is:
\[ \frac{7348.0}{21.33} = 344.5 \text{ cords} \]
**APPENDIX II**

**DERIVATION OF HEIGHTING FORMULA**
(See Figure 11)

\[ f = \text{focal length of camera lens} \]
\[ p = \text{principal point} \]
\[ n = \text{photographic nadir} \]
\[ N = \text{ground nadir} \]
\[ H = \text{flying elevation of camera lens above the base of the object} \]
\[ h = \text{height of object on ground} \]

1) \[ \theta = \text{depression angle of the principal axis from the true horizon, } \arctan Q, \text{ where } Q \text{ is the photo distance from } \frac{f}{p} \text{ the true horizon to the principal point along the principal line.} \]

2) \[ \mu = \arctan m, \text{ where } m \text{ is the } y \text{ coordinate on photo-} \frac{f}{f} \text{graph from the principal parallel to the base of the object. This is negative if the point lies above the principal parallel.} \]

3) \[ \eta = \arctan k, \text{ where } k \text{ is the } y \text{ coordinate, as above, to } \frac{f}{f} \text{ the top of the object.} \]

4) \[ \lambda = \mu - \eta \]

5) \[ \sigma = 90^\circ - \theta - \mu \]

6) \[ \text{Line } LB = r = \frac{H}{\cos \sigma} \]

7) \[ \psi = 180^\circ - \sigma - \lambda \]

8) \[ \frac{h}{\sin \lambda} = \frac{r}{\sin \psi} \]

9) \[ h = \frac{r \sin \lambda}{\sin \psi} \]

Substituting back:

10) \[ h = \frac{H}{\cos \sigma} \sin (\mu - \eta) \]

11) \[ h = \frac{\sin (180^\circ - \sigma - \lambda)}{H \sin (\mu - \eta)} \]

\[ \frac{\cos \sigma \sin (180^\circ - \sigma - \lambda)} {\cos \sigma \sin (180^\circ - \sigma - \lambda)} \]
12) \[ h = \frac{H \sin (\mu - \eta)}{\cos (90^\circ - \theta - \mu) \sin [180^\circ - (90^\circ - \theta - \mu) - (\mu - \eta)]} \]

Substituting:
\[ t = 90^\circ - \theta \]

13) \[ h = \frac{H \sin (\mu - \eta)}{\cos (t - \mu) \sin [180^\circ - (t - \mu) - (\mu - \eta)]} \]

14) \[ h = \frac{H \sin (\mu - \eta)}{\cos (t - \mu) \sin (180^\circ - t + \eta)} \]

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