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An Adaptive Simulation Technique For The One-Dimensional Water Flow Equation

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SUMMARY

The pressure head form of the general flow equation for water in a porous medium was numerically solved using a scheme that allowed both the time step (Δt) and the space increment (Δz) to be changed during the flow process. The mass balance equation was used to check the accuracy of the simulation. If a predefined accuracy criterion was not met, the time step as well as the space increments between a fixed number of gridpoints were changed. The model concentrates a large number of gridpoints in regions of large changes in pressure head gradients. As the number of gridpoints is fixed, fewer grid-points are distributed in regions where small changes occur. The method is demonstrated for an infiltration, drainage, and evaporation process.

AN ADAPTIVE SIMULATION TECHNIQUE FOR THE ONE-DIMENSIONAL WATER FLOW EQUATION

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INTRODUCTION

Many authors have numerically solved the one-dimensional pressure head form of the general flow equation for water in porous media. A comparison of the more popular procedures was presented by Haverkamp et al. (1977). In most cases presented in the literature, a fixed Δz value is used in the spatial domain. An adaptive numerical scheme to solve the pressure head form of the general flow equation was recently published by Dane and Mathis (1981). The difference between their model and most other numerical procedures is that both time and space increments are allowed to change during the simulation. This allows the use of a fine grid in those portions of the flow region known to contain large changes in pressure head gradients. The fine grid spacing moves up and down the soil profile, corresponding with those portions where large changes in pressure head gradient occur. Furthermore, their proposed scheme is applicable to non-steady flow conditions as well. In this report, the source text, together with a description and discussion of the model, is presented.

THEORY

The general transport equation for water movement in the soil which was used in the simulation described in this report can be formulated by combining

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the Darcy equation and the mass balance equation of soil water. In one-dimensional form the Darcy equation may be written as:

$$v = -K(h)\frac{\partial H}{\partial z} \quad , \qquad [1]$$

where K(h) is the hydraulic conductivity function, h is soil water pressure head, H is hydraulic head, z is distance (z=0 at reference level and z > 0 above reference level), and $\frac{\partial H}{\partial z}$ is the hydraulic gradient. The algebraic sign of the flux v indicates the direction of the flow, i.e., v is upward if positive and downward if negative. Since the hydraulic head is the sum of the pressure and gravitational head, Eq. [1] may be written as:

$$v = -K(h)\left[\frac{\partial h}{\partial z} + 1\right] \quad , \qquad [2]$$

where $\frac{\partial h}{\partial z}$ is the pressure head gradient.

For a certain set of assumptions the mass balance equation can be written as a volume balance equation:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial v}{\partial z} \quad . \qquad [3]$$

This equation relates the time rate of change of water content, $\frac{\partial \theta}{\partial t}$, of a differential volume element of soil to the difference of inflow and outflow across that element, $\frac{\partial v}{\partial z}$, which is called the divergence of the flux. Combining Eq. [2] with Eq. [3] yields the general equation for vertical flow of water in soil:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z} + 1\right)\right] \quad . \qquad [4]$$

To obtain an equation for water flow in one dependent variable, another relation between θ and h is required. Introducing the water capacity of the soil as $C(h) = \frac{d\theta}{dh}$ (i.e., the slope of the water retention function), the time derivative term in Eq. [4] can be transformed to:

$$\frac{\partial\theta}{\partial t} = \frac{d\theta}{dh} \cdot \frac{\partial h}{\partial t} = C(h) \cdot \frac{\partial h}{\partial t} \quad . \qquad [5]$$

Substituting Eq. [5] in Eq. [4] yields:

$$C(h) \cdot \frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z} + 1\right)\right] \quad , \qquad [6]$$

which is called the pressure head form of the one-dimensional general flow equation. Equation [6] can be expressed as a system of two first order non-linear differential equations with h(z,t) and v(z,t) being the unknowns:

$$v(z,t) = -K(h)\left(\frac{\partial h}{\partial z} + 1\right) \qquad [7a]$$

and

$$C(h)\,\frac{\partial h}{\partial t} = -\frac{\partial v}{\partial z} \quad , \qquad [7b]$$

for $-L \leq z \leq 0$ and $t > 0$, where v(z,t) is the Darcy flux, and L is the depth of the profile under consideration.

An adaptive numerical method to solve this set of equations, subjected to certain initial and boundary conditions, will be presented.

NUMERICAL IMPLEMENTATION

The solution of Eq. [7] by a finite difference technique requires that a grid be superimposed upon the time (t)-depth (z) region of the flow system under consideration. The independent variables t and z will be subscripted with j and i, respectively.

For $-L \leq z \leq 0$ and $0 \leq t \leq T$, the gridpoints (z_i, t_j) are defined for $i=0,1,\ldots,N$ and $j=0,1,\ldots,M$ such that $z_0=0$, $z_N=-L$, $t_0=0$, and $t_M=T$ (see figure 1). The grid spacing may be arbitrary and is denoted by $\Delta z_i = z_i - z_{i-1}$ and $\Delta t_j = t_j - t_{j-1}$, i.e., both time and space increments may be of nonuniform size. However, the number of gridpoints in the depth direction is fixed.

Space and time derivatives are approximated at time t_j (unknown time level) and at the midpoint of (z_i, z_{i-1}) using backward differences. Both K and C are evaluated at $\frac{1}{2}(h_{i,j} + h_{i-1,j})$.

For $i=1,\ldots,N$ and $j=1,\ldots,M$ we wish to find $h_{i,j}$ and $v_{i,j}$ such that:

$$-K \cdot \left[\frac{h_{i,j} - h_{i-1,j}}{\Delta z_i} + 1\right] = \tfrac{1}{2}(v_{i,j} + v_{i-1,j}) \qquad [8a]$$

and

$$\frac{C}{2\Delta t_j}\left[h_{i,j} + h_{i-1,j} - h_{i,j-1} - h_{i-1,j-1}\right] = -\left[\frac{v_{i,j} - v_{i-1,j}}{\Delta z_i}\right] \qquad [8b]$$

where:

$$K = K\left(\frac{h_{i,j} + h_{i-1,j}}{2}\right)$$

and

$$C = C\left(\frac{h_{i,j} + h_{i-1,j}}{2}\right) .$$

This method is implicit for which, at each time step, a system of (2N+2) nonlinear equations must be solved.

To solve Eq. [8] one needs to define one set of initial conditions and two sets of boundary conditions. The initial condition for a given problem is specified by assigning a pressure head value to each node at t=0:

$$h(z,0) = g(z) \qquad -L \leq z \leq 0 . \qquad [9a]$$

This initial condition yields, according to Eq. [7a], the initial flux values:

$$v(z,0) = -K[g(z)] \cdot \left[\frac{\partial g(z)}{\partial z} + 1\right] . \qquad [9b]$$

Boundary conditions are defined for the top and bottom nodal point for $t > 0$ and are be expressed by:

$$\alpha_1 \, h(0,t) + \alpha_2 \, v(0,t) = g_0(t) \qquad z=0 \qquad [10a]$$

$$\beta_1 \, h(-L,t) + \beta_2 \, v(-L,t) = g\ (t) \qquad z=-L . \qquad [10b]$$

The advantage of expressing the boundary conditions in this manner is the ease with which one can switch from a pressure head to a flux boundary condition and vice versa by assigning the values 0 or 1 to α and β. It should also be noted that the functions g_0 and g_1 are time dependent. Expressed in terms of gridpoints, Eq. [9] and Eq. [10] can be written as:

$h_{i,0} = g(z_i)$

$$v_{i,0} = -K[g(z_i)]\left[\frac{\partial g(z_i)}{\partial z} + 1\right] \qquad [11a]$$

and

$$\alpha_1 h_{0,j} + \alpha_2 v_{0,j} = g_0(t_j)$$

$$\beta_1 h_{N,j} + \beta_2 v_{N,j} = g_1(t_j) \quad . \qquad [11b]$$

Eq. [8] can be solved by the following iterative procedure (see figure 2 also). Suppose $h_{i,j-1}$ and $v_{i,j-1}$ are known for i=0,1,...,N. If H2 and V2 are defined as the unknown h and v at time j, respectively, Eq. [8a] can then be written as:

$$-K\left[\frac{H2(i) - H2(i-1)}{\Delta z_i} + 1\right] = \tfrac{1}{2}[V2(i) + V2(i-1)] \quad .$$

Let H1(i), i=0,...,N be the known pressure heads at time j-1. Equation [8b] can then be approximated by:

$$\frac{C}{2\Delta t_j}[H2(i) + H2(i-1) - H1(i) - H1(i-1)] = -\left[\frac{V2(i) - V2(i-1)}{\Delta z_i}\right] \quad .$$

Grouping the unknowns H2 and V2 yields:

$$-K\cdot H2(i-1) + \tfrac{1}{2}\Delta z_i\cdot V2(i-1) + K\cdot H2(i) + \tfrac{1}{2}\Delta z_i\cdot V2(i) = -K\Delta z_i \qquad [12a]$$

and

$$-\tfrac{1}{2}C \cdot R \cdot H2(i-1) + V2(i-1) - \tfrac{1}{2}C \cdot R \cdot H2(i) - V2(i) = -\tfrac{1}{2}C \cdot R \cdot H1^* \qquad [12b]$$

where $R = \frac{\Delta z_i}{\Delta t_j}$ and $H1^* = H1(i) + H1(i-1)$.

During the first iteration of each time step j, K and C are calculated from the known pressure head values H1 at time j-1, while during subsequent iterations (all during the same time step), K and C are determined from the latest values of H2. If for all depths both H2(i) and V2(i), i=0,...,N differ less than a certain small amount from their corresponding values during the previous iteration, then the latest values for H2(i) and V2(i) are accepted as the true values. If the difference between the previous and the latest computed pressure heads and fluxes is larger than that small amount, the solution process is repeated as before. Consequently, Eq. [12] represents a set of linear algebraic equations in the unknowns H2(i) and V2(i) for i=0,...,N with H2(0), H2(N), V2(0), and V2(N) satisfying the boundary conditions given in Eq. [11b]. Keller (1974) showed that H2(i) and V2(i) will converge to the solution of Eq. [8], provided that the pressure head and flux values over a time step are suitably close.

At a certain time j, this system of linear equations can be put in the matrix form Ax=B, as shown in figure 3. In the simulation program, an IMSL (International Mathematical and Statistical Library) subroutine is used to solve this linear system. However, this subroutine requires that the coefficient matrix A is stored in band storage mode to minimize memory requirements.

A n by n matrix with k lower codiagonals and l upper codiagonals, stored in band storage mode, is reduced to a matrix of dimensions n by (k+l+1). The matrix is stored row wise so that the zero-elements are compressed out of the matrix and the main diagonal elements fall in column (k+1). The 2N+2 by 2N+2 coefficient matrix A, with two upper and lower codiagonals, can be stored in band storage mode as shown below:

$$
\overset{\longleftarrow \text{ 5 columns } \longrightarrow}{}
$$

$$
(2N+2)\text{ rows}\quad
\begin{array}{ccccc}
0 & 0 & \alpha_1 & \alpha_2 & 0 \\
0 & -K & \frac{1}{2}\Delta z_1 & K & \frac{1}{2}\Delta z_1 \\
-\frac{1}{2}C\cdot R & 1 & -\frac{1}{2}C\cdot R & -1 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & -K & \frac{1}{2}\Delta z_N & K & \frac{1}{2}\Delta z_N \\
-\frac{1}{2}C\cdot R & 1 & -\frac{1}{2}C\cdot R & -1 & 0 \\
0 & \beta_1 & \beta_2 & 0 & 0
\end{array}
$$

The solution process uses the initial condition (Eq. [11]) to solve for the pressure heads and fluxes at the end of the first time step. This set of solutions is then used to solve for the next time step. The solution process thus marches forward in time by increments Δt_j.

CALCULATION OF THE TIME STEP

The mass balance equation was used to control the time step size. A too large time step will result in an inaccurate approximation of $h_{i,j}$ and $v_{i,j}$ (see Eq. [8]), which in turn will influence the outcome of the mass balance equation. At time t_j, the mass balance MB_j is defined as:

$$MB_j = \left| \int_{-L}^{0} [\theta(z,t_j) - \theta(z,t_{j-1})]dz - \int_{t_{j-1}}^{t_j} [v(0,t) - v(-L,t)]dt \right|, \quad [13]$$

where $\theta(z,t)$ is a function of pressure head and can thus be calculated. The increase in water in a profile (first integral form in Eq. [13]) was estimated by applying the trapezoidal rule. The second integral is simply a subtraction of the fluxes at the top and bottom of the profile under consideration and integrated over Δt_j. If MB_j was larger than a specific value ε, the values of $h_{i,j}$ and $v_{i,j}$ were rejected, the time step decreased, and new values for $h_{i,j}$ and $v_{i,j}$ were calculated. Very small values for MB_j, e.g. 0.1ε, resulted in an increase of the time step.

A relative mass balance was also calculated after each time step. The relative mass balance (percent) is defined as MB_j x 100 divided by the total amount of water that enters (or leaves) the profile across the bottom and top boundary over a given time step, while the overall relative mass balance (percent) is defined similarly but over the total time of simulation. The relative mass balance may be a better tool to check accuracy, especially if the second term of the right hand side of Eq. [13] is small relative to MB_j.

CALCULATION OF THE SPACE STEP

Space steps between the various gridpoints are nonuniform and may be changed during the simulation. The space steps Δz_i are estimated in the following way.

The space derivative $\frac{\partial h}{\partial z}$ is approximated exactly if h were a piecewise linear function of z. A function F(z) is piecewise linear with respect to the grid z_i if F(z) is continous for $-L < z < 0$ and linear over the interval $z_i < z < z_{i-1}$, where i=1,2,...,N. However, $h(z,t_j)$ is in most cases not piecewise linear, hence errors are made if space derivatives are approximated by difference equations. To reduce this error as much as possible, the z_i's are selected in such a way that they will determine the best possible piecewise interpolation of pressure head at t_j, using N+1 gridpoints. This is accomplished if

$$\max_{z_{i-1} \le z \le z_i} \left| \frac{\partial^2 h(z,t_j)}{\partial z^2} \right| \Delta z_i^{\,2}$$

is constant for each space step z_i. Hence, at time j, a function G_j is defined as

$$G_j = \sqrt{\left|\frac{\partial^2 h(z,t_j)}{\partial z^2}\right|} \cdot \Delta z_1 = \ldots = \sqrt{\left|\frac{\partial^2 h(z,t_j)}{\partial z^2}\right|} \cdot \Delta z_i = \sqrt{\left|\frac{\partial^2 h(z,t_j)}{\partial z^2}\right|} \cdot \Delta z_N \qquad [14]$$

for i=1,...,N.

The function G_j is calculated from:

$$G_j = \frac{1}{N} \sum_{i=1}^{N} \sqrt{\left|\frac{\partial^2 h(z,t_{j-1})}{\partial z^2}\right|} \Delta z_i^* \qquad \text{for} \qquad i=1,\ldots,N$$

where Δz_i^* refers to time t_{j-1}. In other words, the space steps Δz_i^*, i=1,...,N do not satisfy Eq. [14]. More details about the above procedure are included in Appendix I.

The second derivative of pressure head with respect to z can be approximated by direct application of Darcy's law:

$$v = -K\frac{\partial H}{\partial z} = -K\frac{\partial h}{\partial z} - K$$

and

$$\frac{\partial v}{\partial z} = -\frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right) - \frac{\partial K}{\partial z} = -\frac{\partial K}{\partial z} \cdot \frac{\partial h}{\partial z} - K\frac{\partial^2 h}{\partial z^2} - \frac{\partial K}{\partial z} .$$

Solving the above equation for $\frac{\partial^2 h}{\partial z^2}$ yields:

$$\frac{\partial^2 h}{\partial z^2} = \left[- \frac{\partial v}{\partial z} - \frac{\partial K}{\partial z} \left(\frac{\partial h}{\partial z} + 1 \right) \right] / K \quad ,$$

which can be approximated by:

$$\left[- \left(\frac{v_{i,j-1} - v_{i-1,j-1}}{\Delta z_i} \right) - \left(\frac{K(h_{i,j-1}) - K(h_{i-1,j-1})}{\Delta z_i} \right) \left(\frac{h_{i,j-1} - h_{i-1,j-1}}{\Delta z_i} + 1 \right) \right] / K^*$$

where $K^* = K \left(\frac{h_{i,j-1} + h_{i-1,j-1}}{2} \right)$.

Conforming to the requirement set by Eq. [14], new z_i's are calculated, whereas the number of space steps N remains the same. However, if the values of z_i at time t_j are changed, new values for $h_{i,j-1}$ and $v_{i,j-1}$ must also be calculated. This can be accomplished by piecewise linear interpolation of $h_{i,j-1}$ and $v_{i,j-1}$.

Figure 4 shows an infiltration profile where theta and the first and second derivative of the pressure head with respect to depth are plotted against depth. The smallest space increments will occur at depths with a relatively large absolute value for the second derivative (see also Eq. [14]). For this particular example, an infiltration case, Figure 4 shows that the highest density of gridpoints occurs in the vicinity of the wetting front. Since the water content below the toe of the wetting front has not yet changed, very few gridpoints are needed in this region.

To save computer time, Δz_i was changed only if Δt_j was changed, according to the size of MB_j.

RESULTS

Three examples will be presented in this section. The first concerns a drainage profile of which the results are compared with those obtained with UNSAT1, a simulation model published by Van Genuchten (1978). The second example describes the infiltration of water into the soil as reported by Haverkamp et al. (1977). In the third example problem, published by Klute and Heermann (1978), evaporation was simulated.

1. Drainage from a Uniform Soil Profile.

Drainage is allowed to occur in a uniform 1.4-m deep soil profile. The following expressions were used to describe the soil hydraulic properties of the soil (Van Genuchten, 1978):

$$\theta = \theta_r + \left[\frac{\theta_s - \theta_r}{\left[1 + (\alpha|h|)^n\right]^m}\right] .$$

and

$$K = K_s \Theta^{\frac{1}{2}} [1 - (1 - \Theta^{\frac{1}{m}})^m]^2 ,$$

where θ_r and θ_s represent the residual and saturated water contents respectively; α, n, and m $(= 1 - \frac{1}{n})$ are parameters characteristic of the particular soil, K_s is the saturated hydraulic conductivity, and Θ is the dimensionless soil water content, defined as:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} .$$

The soil used was a Troup loamy sand (Dane, 1980) for which the following parameter values apply:

$K_s = 10.95$ cm/hr $\quad\quad \alpha = 0.02912$

$\theta_s = 0.365$ $\quad\quad n = 3.57168$

$\theta_r = 0.069$.

The following initial and boundary conditions were adopted:

$$h(z,0) = -26.774\text{cm}$$

or

$$\theta = 0.30 \qquad 0 \leq L \leq 140 \text{ cm} \qquad t=0$$

$$-K\left(\frac{\partial h}{\partial z}+1\right) = 0 \text{ (zero flux)} \qquad z=0 \qquad t \geq 0$$

$$\frac{\partial h}{\partial z} = 0 \text{ (unit hydraulic gradient)} \qquad z=-140 \text{ cm} \qquad t \geq 0 \text{ .}$$

The results of this example problem were compared with results obtained from a simulation model published by Van Genuchten (1978), who used a Hermitian finite element scheme.

Table 1 shows that no substantial differences occurred between the two methods. However, there is an important distinction between the two, as is shown in figure 5. When theta is plotted against depth, the described model, WAFLOW, concentrates its gridpoints in regions where the largest changes occur, whereas UNSAT1 (Van Genuchten's model) uses a constant space step between all gridpoints.

2. Steady State Infiltration into a Sandy Profile.

In this example (Haverkamp et al., 1977) water is allowed to infiltrate into a 70-cm deep homogeneous sandy soil profile having the following hydraulic properties:

$$K = K_s \frac{A}{A+|h|^{\beta}}, \qquad \begin{aligned} K_s &= 34 \text{ cm/hr.} \\ A &= 1.175 * 10^6 \\ \beta &= 4.74 \end{aligned}$$

and

$$\theta = \frac{\alpha(\theta_s - \theta_r)}{\alpha + |h|^{\beta}} + \theta_r, \qquad \begin{aligned} \theta_s &= 0.287 \\ \theta_r &= 0.075 \\ \alpha &= 1.611 * 10^6 \\ \beta &= 3.96 \end{aligned}$$

The initial and boundary conditions for infiltration of water in sand were:

$h = -61.5$cm	$0 \leq L \leq 70$cm	$t=0$
$v = -13.69$ cm/hr.	$z=0$	$t \geq 0$
$h = -61.5$ cm	$z=-70$ cm	$t > 0$,

where v denotes a constant downward flux and the bottom boundary condition a constant pressure head. Figure 6 shows the water content profiles after 0.2 and 0.4 hr. simulation time (using 71 gridpoints) and also the redistribution of the gridpoints as the wetting front moves down. The smallest space steps are at the wetting front where the largest values of $\partial^2 h/\partial z^2$ occur. The results compared favorably with those published by Haverkamp et al. (1977) for the same soil characteristics.

Table 2 reflects the effect of the number of space steps on computer time and accuracy. It is interesting that the accuracy, expressed in absolute and relative mass balance, is markedly reduced only when the number of space steps decreases below 20. However, the required computer time is greatly reduced.

3. Evaporation and Drainage in a Uniform Soil Profile with Fixed Water Table

As was shown by Dane and Mathis (1981), evaporation can also be simulated with the model. Their example problem was published by Klute and Heermann (1978), who simulated evaporation and drainage in uniform, coarse, uranium mill tailings with a fixed water table at 3 m depth. The hydraulic properties of the tailings were represented by the following empirical functions:

$$\theta(h) = \theta_o \left[\frac{\cosh\beta - \Gamma}{\cosh\beta + \Gamma}\right] \qquad -7320 \leq h \leq 0 \text{ cm}$$

$$\theta(h) = \left(\frac{h}{d}\right)^a \qquad h < -7320 \text{ cm}$$

$$K(\theta) = Ae^{B\theta}$$

$$\beta = \left(\frac{h}{h_o}\right)^b \qquad b < 0$$

and

$$\Gamma = \frac{\theta_o - \theta_r}{\theta_o + \theta_r} \quad ,$$

where θ_o, θ_r, h_o, b, and d are parameters which determine the function of the initial drainage curve. The following values for the parameters of the functions were given:

$\theta_o = 0.433$ $\quad d = -0.0171$

$\theta_r = 0.076$ $\quad a = -0.1964$

$h_o = -131$ $\quad A = 0.225*10^{-8}$

$b = -0.468$ $\quad B = 54.29$.

The simulation of drainage with evaporation was subjected to the conditions:

$h = -1$ cm $\quad t=0 \quad -300 \leq z \leq 0$

$q = 1/24$ cm/hr $\quad t > 0 \quad z=0 \quad h > -15000$ cm

$h = 0$ cm $\quad t > 0 \quad z=-300$ cm .

At the surface, a constant flux condition was changed to a constant pressure head condition when the pressure head reached a value of -15,000 cm water. Figure 7 shows an evaporation profile. Again, the data points not only indicate the water content, but also the space step distribution. The obtained results were in close agreement with those published by Klute and Heermann.

DISCUSSION

The aspect of the number of gridpoints required to simulate a particular flow problem has not been considered in this communication, but it will largely depend on the type of problem and the depth of the soil profile. As shown in table 2, reducing the number of gridpoints will greatly reduce computing time. It might also be possible to let the model itself determine the number of gridpoints required. This can probably be done by assigning a criterion value to G_j in Eq. [14]. Values of G_j above that criterion would then result in an increase of gridpoints and vice versa. Values of pressure heads and fluxes could then be found by piecewise linear interpolation between the old gridpoint depths.

Table 1 shows a relatively large computing time for the model presented in this report. However, by increasing the mass balance criterion ε, which was originally set to 0.001 cm, the amount of computing time decreased considerably. Different values for ε, with their respective computing times and mass balance values, are listed in table 3 for the first example problem. As expected, the accuracy of the solution decreased with larger values for ε.

Furthermore, the convergence criterion requires relatively small time steps, as the differences in values for the flux and pressure head at two consecutive time steps must be small.

The simulation model presented in this report is applicable only to uniform soils. However, it should be possible to extend the model to flow problems in layered soils.

The uniqueness of the model is its ability to redistribute the space steps whenever this is necessary, i.e., when large changes in pressure head gradients occur. It, therefore, eliminates manual redistribution of gridpoints and makes computer use more efficient.

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APPENDIX I

APPROXIMATION OF SPACE STEPS

For a choice of Δz_i, the following procedure was used. First note that the difference equations approximate the space derivatives exactly if h and v (Eq. [8]) are piecewise linear functions of z. For t_j and N fixed, let $h(z,t_j)$ be the exact (unknown) solution to the boundary value problem given in Eq. [7]. We will assume that $h(z,t_j)$ has a continuous second partial derivative with respect to z. Then the z_i's are selected such that, if F(z) is the piecewise linear function with $F(z_i)=h(z_i,t_j)$, for i=0,1,...,N, the error, $\max|F(z)-h(z,t_j)|$, $-L \le z \le 0$, is as small as possible. That is, the z_i's we wish to use are those which will uniquely determine the best possible piecewise interpolation of the pressure head at t_j using N+1 gridpoints. This is accomplished if

$$\max_{z_{i-1} \le z \le z_i} \left| \frac{\partial^2 h(z,t_j)}{\partial z^2} \right| \cdot \Delta z_i^2$$

is constant for each i=1,2,3,....,N. Although this is, in general, a difficult problem in nonlinear programming, we may obtain a reasonable approximation of the z_i's similarly as in the algorithm NEWNOT (page 184, de Boor, 1978). Let

$$g(z) = \left\{ \left| \frac{\partial^2 h(z,t_j)}{\partial z^2} \right| \right\}^{\frac{1}{2}}$$

and

$$\int_{z_0}^{z_N} g(s)ds = \sum_{i=1}^{N} \int_{z_{i-1}}^{z_i} g(s)ds \ .$$

Since $\int_{z_{i-1}}^{z_i} g(s)ds$ is approximately equal to

$$\left\{\max_{z_{i-1} \le z \le z_i} g(z)\right\} \cdot \Delta z_i ,$$

we may select the z_i so that

$$\int_{z_{i-1}}^{z_i} g(s)ds = \frac{1}{N} \int_{z_0}^{z} g(s)ds \quad .$$

To accomplish this, define

$$G(z) = \int_{z_0}^{z} g(s)ds$$

and assume that $g(s) \neq 0$. Then G is monotone increasing and so has an inverse G^{-1}. Since

$$G(z_N) = \int_{z_0}^{z_N} g(s)ds = N \int_{z_{i-1}}^{z_i} g(s)ds ,$$

$$G(z_i) = \int_{z_0}^{z_i} g(s)ds = \frac{i}{N} G(z_N) \quad .$$

Hence:

$$z_i = G^{-1}\left\{\frac{i}{N} G(z_n)\right\}, \; i=1,2,\ldots,N.$$

The following example illustrates the selection of z_i. Suppose $h(z)=z^3$, $0 \le z \le 1$, and $N=2$ so that $z_0=0$, $z_2=1$, and $z_1=a$ is to be determined. If a is fixed and a piecewise linear function F(z) is constructed, it will look like the dashed line (Appendix figure 1). Approximation theory tells us that the error of approximating h(z) with F(z) is less than

$$\max_{0 \leq z \leq a} \frac{|h''(z)|}{8} \cdot \Delta z_1^2 = \frac{6a}{8} \cdot a^2$$

in the first interval and less than

$$\max_{a \leq z \leq 1} \frac{|h''(z)|}{8} \cdot \Delta z_2^2 = \frac{6}{8} (1-a)^2$$

in the second interval. Hence, the overall error will be less than

$$\max \left\{ \frac{6a^3}{8}, \frac{6}{8} (1-a)^2 \right\} .$$

The best choice of a is the one that makes

$$\max \left\{ \frac{6a^3}{8}, \frac{6}{8} (1-a)^2 \right\}$$

as small as possible. We claim that this is accomplished if the two errors are equal. That is

$$\frac{6}{8} a^3 = \frac{6}{8} (1-a)^2 \quad \text{or} \quad a = 0.56984 .$$

To see that this is true, let a_* be the solution of

$$\frac{6}{8} a^3 = \frac{6}{8} (1-a)^2$$

and

$$\max\left\{\frac{6}{8}a_*^3, \frac{6}{8}(1-a_*)^2\right\} = \frac{6}{8}a_*^3 = \frac{6}{8}(1-a_*)^2 .$$

Now if $a<a_*$, then

$$\max\left\{\frac{6a^3}{8}, \frac{6}{8}(1-a)^2\right\} = \frac{6a^3}{8} > \frac{6a_*^3}{8}$$

and the error is larger than using a_*.

Likewise if $a < a_*$,

$$\max\left\{\frac{6a^3}{8}, \frac{6}{8}(1-a)^2\right\} = \frac{6}{8}(1-a)^2 > \frac{6}{8}(1-a_*)^2$$

and again the error is larger. The smallest error is therefore obtained by using a_*!

In general, however, we do not know $h''(z)$ exactly, but suppose we have approximations, for example

$h''(z) \approx 1.5 \quad 0 \le z \le 0.5$

$h''(z) \approx 4.5 \quad 0.5 \le z \le 1.$

Now:

$$\frac{|h''(z_1)|}{8} \cdot \Delta z_1^2 = \frac{|h''(z_2)|}{8} \cdot \Delta z_2^2$$

or

$$\sqrt{|h_1''|} \cdot \Delta z_1 = \sqrt{|h_2''|} \cdot \Delta z_2 .$$

Therefore, the problem of finding a_* may be approximated by finding a_*, so that the two areas in Appendix figure 1 are equal. Assuming $a > 0.5$ then:

$$A_1 = \tfrac{1}{2}\sqrt[4]{1.5} + (a-0.5)\sqrt[4]{4.5}$$

and

$$A_2 = (1-a)\sqrt[4]{4.5} \quad .$$

Solving $A_1 = A_2$ will yield $a = 0.6057$ as an approximation to a_*.

APPENDIX II

EXECUTION OF WAFLOW

Appendix figure 2 shows the flow chart of the simulation model WAFLOW. The program consists of two subroutines (UNSTE and REZEE) and four functions (FK, FC, FTH, and UIN). The input data are read from the data file INPUT1.DATA. The functions FK, FC, and FTH define the hydraulic functions K(h), C(h), and θ(h), respectively, for the soil type under consideration. The function UIN provides the initial conditions, expressed in pressure heads as a function of depth. Upon execution of the program, a listing is printed of the soil's hydraulic properties and the initial conditions. The subroutine UNSTE allows changes in the top and bottom boundary conditions with respect to time. To solve the linear system of differential equations, an IMSL subroutine was used. Depending on the desired accuracy this external subroutine is either LEQT1B or LEQT2B. LEQT2B uses an iterative improvement to obtain a more accurate solution.

If the solution does not satisfy the criterion ε of the mass balance equation, new space steps are generated in the subroutine REZEE. Also the time step is then decreased and the iteration process is repeated. To save computer time, the maximum number of iterations in one time step may not exceed a predefined value IT.

The simulation, on the other hand, proceeds in time if both the convergence and mass balance criteria are met. The time step size will increase for subsequent calculations if the mass balance is less than 1/10 of the imposed criterion. The simulation proceeds in time until either the maximum simulation time is reached (TIM $\geq$ TEND), or a maximum number of time steps is executed (KOUNT $>$ NT).

Appendix table 1 gives a list of the most significant variables used in WAFLOW. Instructions for preparing the input data, together with a listing of actual input data, are given in Appendix table 2. These input data refer to example problem 2, the infiltration profile. A description and listing of the output of this example are given in Appendix table 3, while the listing of the program itself is given in Appendix table 4.

Table 1. Comparison of the Described Model WAFLOW with UNSAT1 for a 1.40-m Deep Drainage Profile Consisting of 29 Gridpoints

	Simulated time (hr.)									
	0.51		5.01		14.42		26.44		50.60	
	WAFLOW	UNSAT1	WAFLOW	UNSAT1	WAFLOW	UNSAT1	WAFLOW	UNSAT1	WAFLOW	UNSAT1
Drainage rate (cm/hr.)......	3.346	3.351	1.348	1.387	0.366	0.379	0.157	0.160	0.065	0.066
Overall absolute mass balance (cm)..	0.0004	0.0002	0.029	0.037	0.055	0.033	0.003	0.0005	0.08	0.005
Moisture in profile (cm)..	40.285	40.287	29.543	29.566	23.081	23.050	20.208	20.159	17.776	17.711
Total CPU (min.)........									2.41	1.37

Table 2. Effect of the Number of Space Steps on Computer Time and Mass Balance during 0.8 hr. of Infiltration in a 70-cm Deep Profile (Mass Balance Criterion ε is 0.001)

Number of space steps NZ	CPU-time (min.)	Relative mass balance (pct.) at time (hr.)			Overall absolute mass balance (cm) at t = 0.8 hr.	Overall relative mass balance (pct.) at t = 0.8 hr.
		0.1	0.4	0.8		
100	4.72	1.14	1.53	0.98	0.13	1.21
70	3.31	0.5	1.53	1.61	0.15	1.33
60	2.86	0.82	1.27	0.85	0.14	1.27
50	2.37	1.14	1.50	1.42	0.14	1.29
40	2.08	1.18	1.50	1.72	0.13	1.20
30	1.64	1.12	1.54	0.91	0.13	1.19
20	1.15	1.04	0.23	1.21	0.12	1.09
15	--	error criterion not met			--	--

Table 3. Effect of Mass Balance Criterion on Computing Time and Mass Balance 65 hr. after Drainage Started

ε (cm)	CPU (min.)	Overall absolute mass balance (cm) at t = 65 hr.	Overall relative mass balance (pct.) at at t = 65 hr.
0.001	2.40	0.0855	0.34
0.005	1.44	0.1957	0.78
0.01	1.15	0.2703	1.07

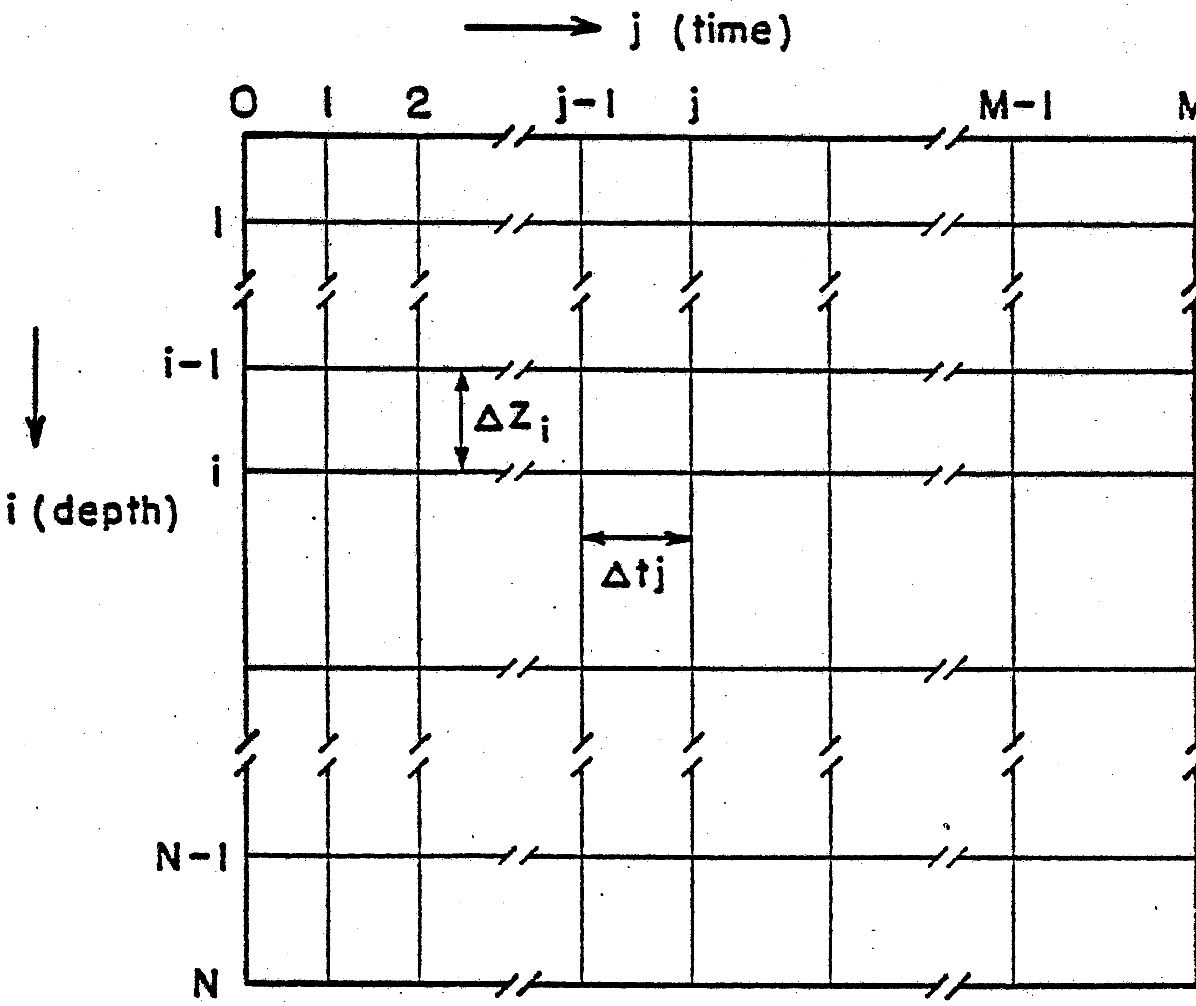

FIG. 1. Diagram showing the finite difference grid superimposed on the depth-time region of a soil profile.

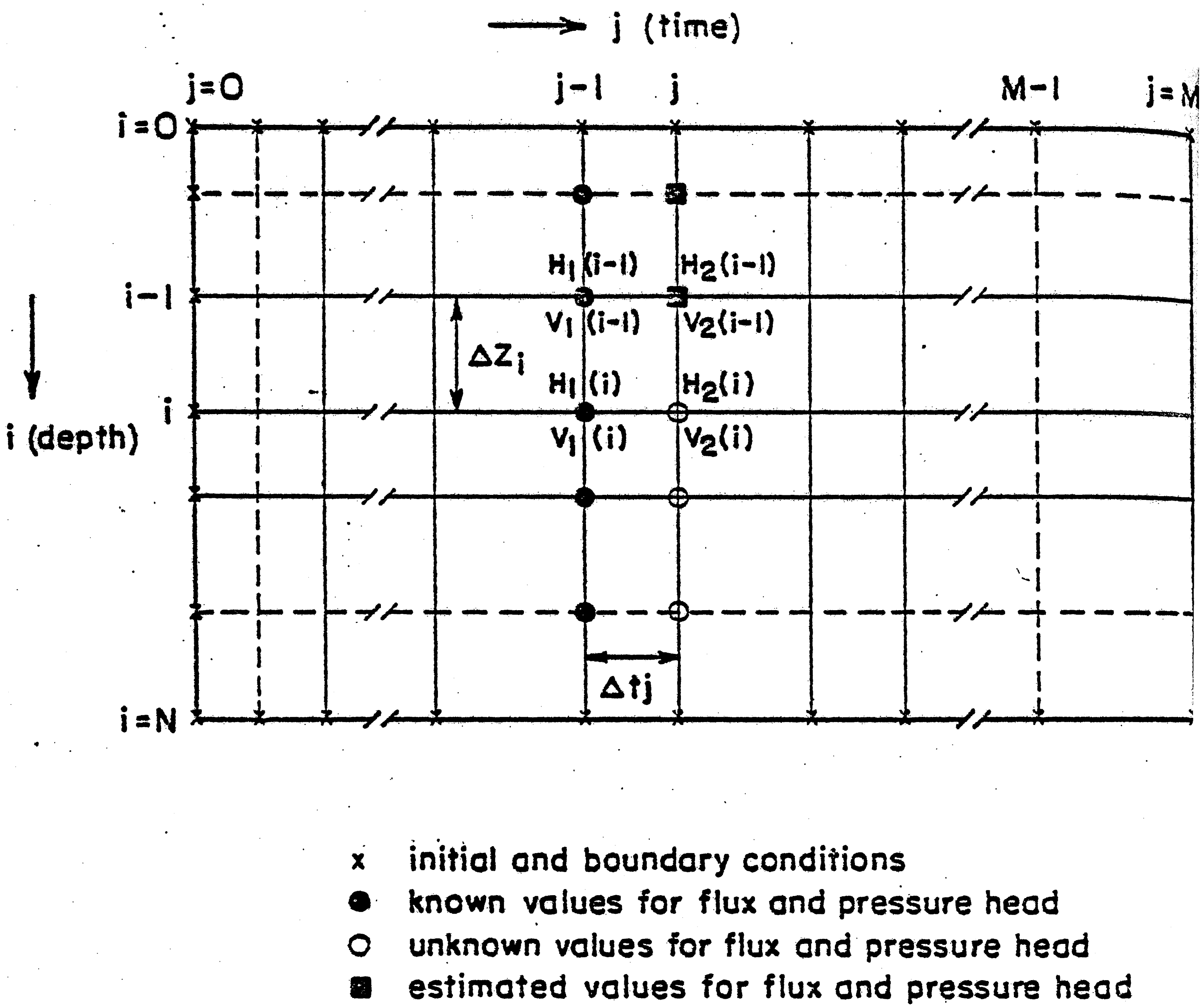

FIG. 2. Depth-time region under consideration about the general gridpoint (i,j) showing the identification of the gridpoint values of pressure head and flux.

$$
\begin{bmatrix}
\alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
-K & \frac{1}{2}\Delta z_1 & K & \frac{1}{2}\Delta z_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2}C\cdot R & 1 & -\frac{1}{2}C\cdot R & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -K & \frac{1}{2}\Delta z_2 & K & \frac{1}{2}\Delta z_2 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2}C\cdot R & 1 & -\frac{1}{2}C\cdot R & -1 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & -K & \frac{1}{2}\Delta z_{N-1} & K & \frac{1}{2}\Delta z_{N-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & -\frac{1}{2}C\cdot R & 1 & -\frac{1}{2}C\cdot R & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -K & \frac{1}{2}\Delta z_N & K & \frac{1}{2}\Delta z_N \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\frac{1}{2}C\cdot R & 1 & -\frac{1}{2}C\cdot R & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \beta_1 & \beta_2
\end{bmatrix}
\times
\begin{bmatrix}
H2(0) \\ V2(0) \\ H2(1) \\ V2(1) \\ \\ \vdots \\ \\ H2(N-1) \\ V2(N-1) \\ H2(N) \\ V2(N)
\end{bmatrix}
=
\begin{bmatrix}
B.C. \\ -K\Delta z_1 \\ -\frac{1}{2}C\cdot R\cdot H1^* \\ \vdots \\ \\ \vdots \\ \\ \vdots \\ -K\Delta z_N \\ -\frac{1}{2}C\cdot R\cdot H1^* \\ B.C.
\end{bmatrix}
$$

Fig. 3. Matrix Form for the System of Linear Equations as Presented in Eq. [12].

The coefficient matrix A has (2N+2) rows and (2N+2) columns. B.C. refers to boundary condition, while all other symbols are explained in the text.

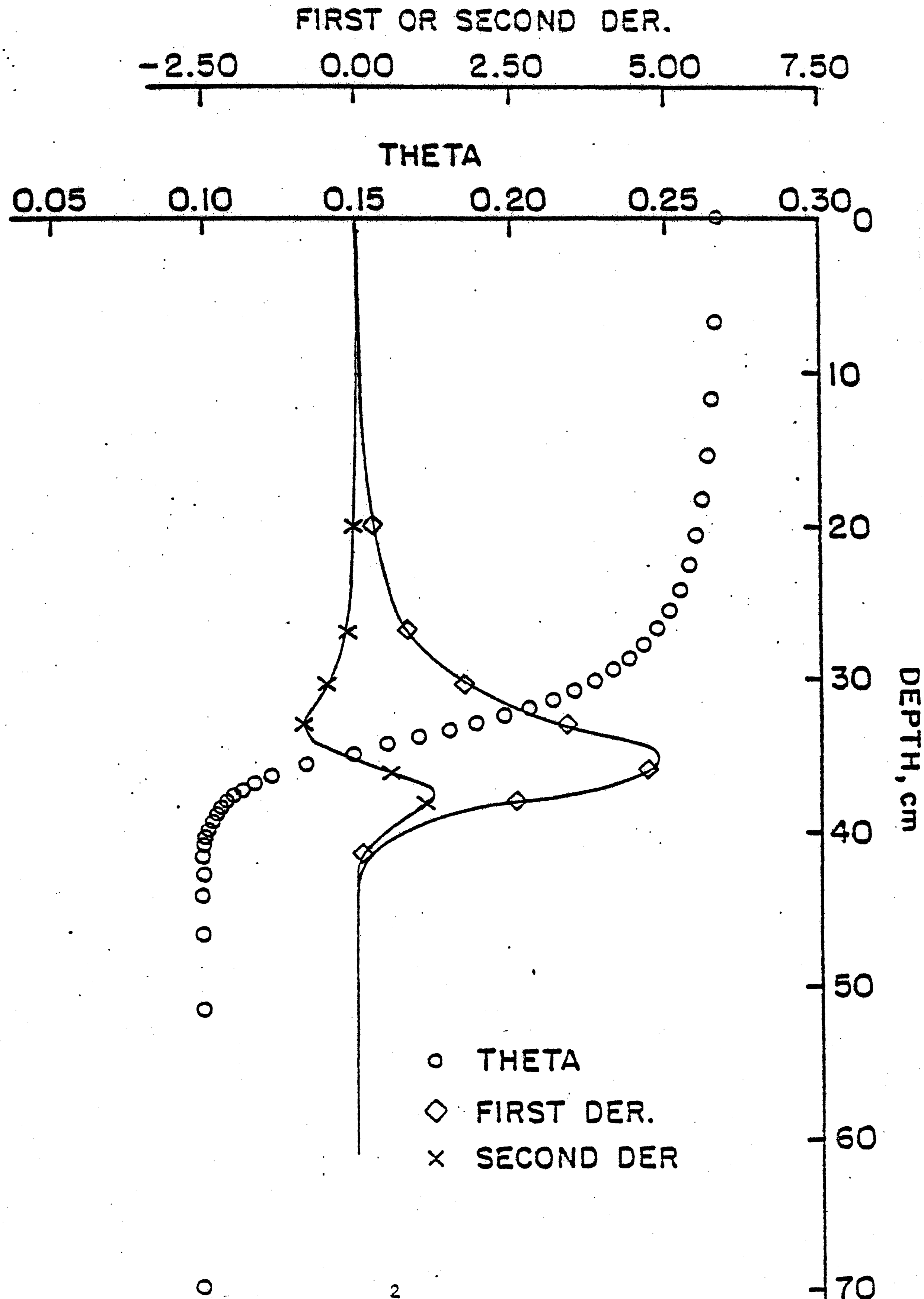

FIG. 4. Water Content, $\frac{\partial h}{\partial z}$, and $\frac{\partial^2 h}{\partial z^2}$ as a Function of Depth During infiltration (41 gridpoints).

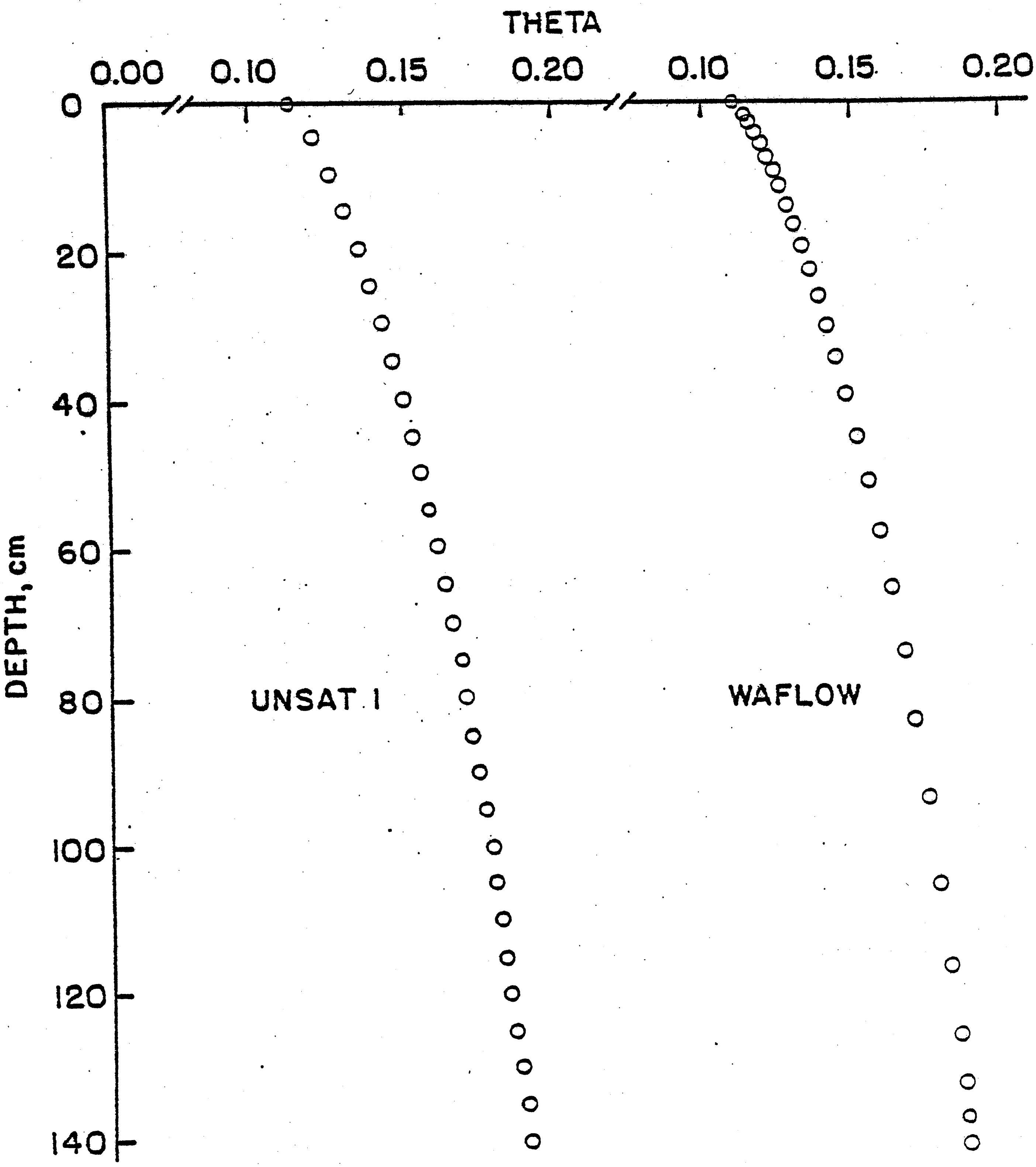

FIG. 5. Water content profiles calculated with the models UNSAT1 and WAFLOW. Number of gridpoints: 29. Time from start of drainage = 14.4 hr. The individual data points indicate the space in the z-direction.

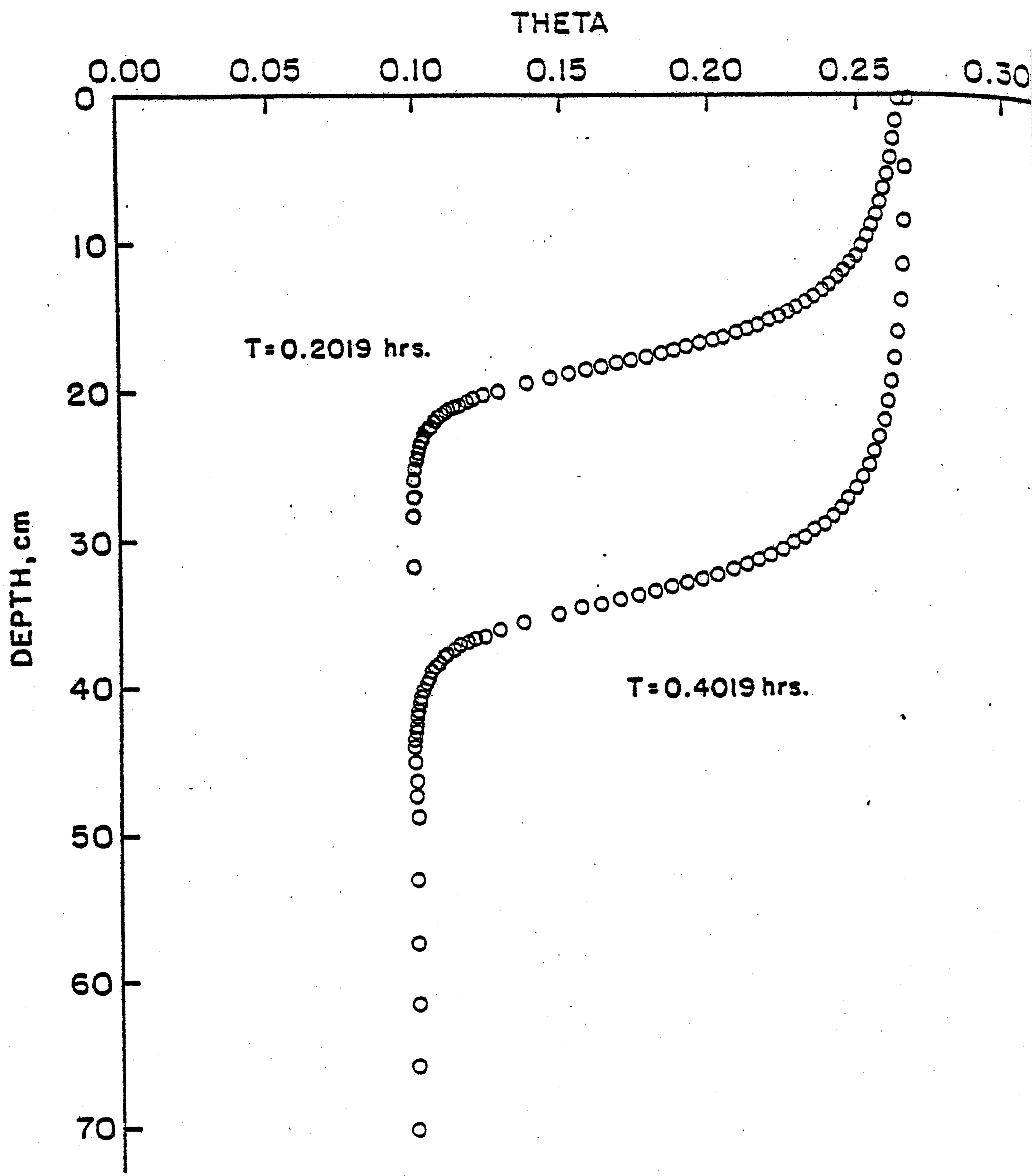

FIG. 6. Water content profiles during infiltration, 0.2019 and 0.4019 hr. since start of simulation. The individual data points indicate the spacing in the z-direction.

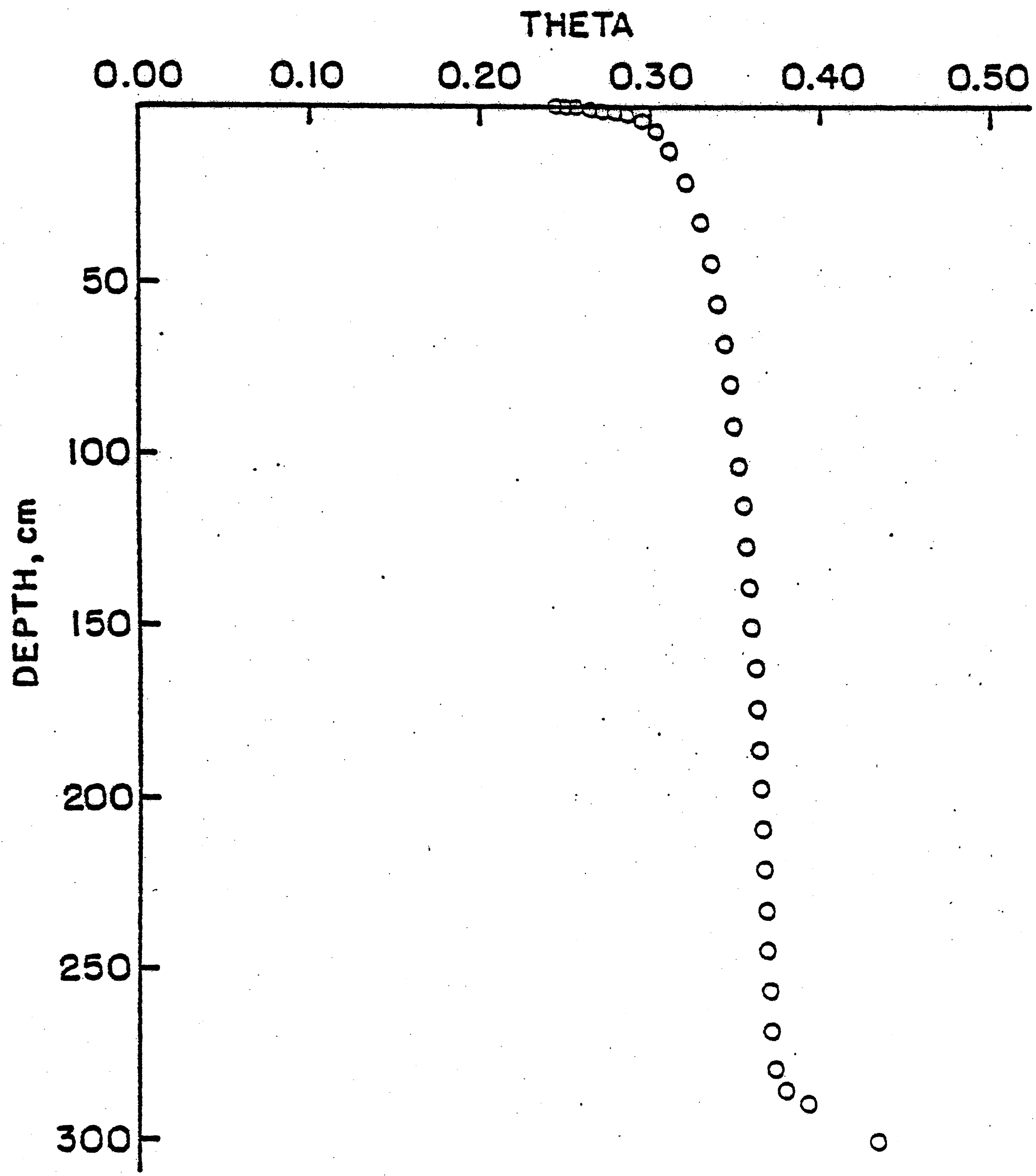

FIG. 7. Water content profile after 4.0106 hr., when evaporation rate is 1/24 cm/hr. The individual data points indicate the spacing in the z-direction.

Appendix Table 1. Definition of the Main Program Variables in WAFLOW.

If the variable represents an array, the maximum dimension of that array is specified.

Variable	Definition
A(310,5)	Coefficient matrix of Ax=B. Input matrix for solution of linear system.
A1(310,7)	Working space for IMSL subroutine LEQT2B.
ALPH1 and ALPH2	Specification whether the top boundary condition is a pressure head or a flux: ALPH1 = 0 and ALPH2 = 1 → constant flux. ALPH1 = 1 and ALPH2 = 0 → constant pressure head.
BETH1 and BETH2	Specification whether the bottom boundary condition is a pressure head or a flux: BETH1 = 0 and BETH2 = 1 → constant flux. BETH2 = 1 and BETH2 = 0 → constant pressure head.
CTNEWS	Increase in stored moisture in one time step (cm).
DEN	Substitute for U2 and V2, used in calculations to check if U2 and V2 meet the criterion for convergence.
DT	Time step (hr.)
DZ	Space step (cm)
EPS	Criterion for the mass balance (cm).
FC	Function, which computes water capacity from pressure head value (cm^{-1}).
FK	Function, which computes hydraulic conductivity from pressure head value (cm/hr.).

Appendix Table 1. (continued)

Variable	Definition
FTH	Function, which computes theta from pressure head value.
IA	Row dimension of A.
IER	Error message from IMSL-subroutine.
IJOB	Optional parameter in argument list of IMSL-subroutine.
IT	Maximum iterations allowed for one time step.
ITIM	Parameter which keeps track of the times when output has to be printed.
IFLAG	Parameter which is initially set equal to 0, indicating that REZEE must be called in the first time step.
IFLOW	Parameter defining the main direction of flow, i.e. infiltration or evaporation: IFLOW = 0 → Evaporation model. IFLOW = 1 → Infiltration model.
ISWIT	Parameter initially set to 0, indicating that REZEE must be called during the time step when the top boundary condition is changed from constant flux to a constant pressure head.
KK	Number of iterations executed in current time step.
KOUNT	Number of time steps executed.
LEQT1(2)B	IMSL-subroutine.
M	Number of columns in RH.
NC	Maximum number of space steps allowed.
NL	Number of lower codiagonals in coefficient matrix A.
NSOIL	Parameter which refers to the soil type.
NT	Total number of time steps allowed in one simulation.

Appendix Table 1. (continued)

Variable	Definition
NTOUT	Number of times that output must be printed.
NZ	Number of space steps.
NZP1	Number of gridpoints, i.e. NZ+1.
NZP4	Last row of coefficient matrix A, i.e. 2*(NZ-1) + 4.
R	Ratio of DZ and DT.
REZEE	Subroutine that calculates new space steps between gridpoints.
RH(310)	Input-output matrix of Ax=B. On input RH contains B, on output X, the unknowns of the linear system.
RMAXK	MAX(X1, X2, RMAXK).
SAVDEN	Summation of SAVE1 since start of simulation (cm).
SAVE1	Increase in stored water over current time step (cm) as calculated from 2 consecutive water content profiles.
SAVE2	Intake of moisture by profile over current time step as calculated from fluxes at the boundaries (cm).
SAVMAS	Summation of SAVE2 since start of simulation (cm).
TEND	End of simulation (hr.).
TH(150)	Theta, volumetric water content.
TIM	Time since start of simulation (hr.).
TM(10)	Array containing the times that output is desired.
U0(150), U1(150), U2(150)	Arrays containing previous, stored, and current values of pressure head for all gridpoints.

Appendix Table 1. (continued)

Variable	Definition
UBOT	Boundary condition at bottom of profile.
UIN	Function, containing the initial pressure head values as a function of depth.
UMAX	Convergence criterion for iterative solution process (0.0005).
UNSTE	Subroutine which may account for transient top or bottom boundary conditions.
US(150)	Array containing the pressure heads at the first iteration of the current time step.
UTOP	Boundary condition at the top of profile under consideration.
VO(150), V1(150), V2(150)	Arrays containing previous, stored, and current flux values for all gridpoints.
VS(150)	Array containing the flux values at the first iteration of the current time step.
WIN	= SAVE2
X1	Absolute value of maximum difference between flux values of first and last iteration in current time step (cm/hr.).
X2	Absolute value of maximum difference between pressure head values of first and last iteration in current time step (cm).
XL(2100)	Work space for IMSL-subroutine.
XMB	Absolute mass balance at current time step (cm).
XMB1	Relative mass balance at current time step (%).
Z(150)	Array containing the depths of all gridpoints.
ZBOT	Depth of profile (cm).

Appendix Table 2. Required Input Data and a Listing of Actual Input Data for Example Problem 2 (Infiltration Profile)

1. Input data file: INPUT1.DATA

COLUMN	FORMAT	VARIABLE	DESCRIPTION
1-5	I5	IFLOW	= 0: evaporation at top boundary. = 1: infiltration or zero flux at top boundary.
6-10	I5	NZ	Number of space steps.
11-15	I5	NT	Maximum number of time steps.
16-20	I5	NTOUT	Number of times output is printed.
21-25	I5	NSOIL	Soil type.
1-10	F10.5	ZBOT	Depth of profile under consideration (cm).
11-20	F10.5	UTOP	Value for top boundary condition: Negative pressure head - negative value (cm). Positive pressure head - positive value (cm). Downward flux - negative value (cm/hr.). Upward flux - positive value (cm/hr.).
21-30	F10.5	UBOT	Value for bottom boundary condition.
31-40	F10.5	DT	Initial time step (hr.).
41-50	F10.5	TEND	Maximum simulation time (hr.).
1-5	F5.2	ALPH1	See table 4.
6-10	F5.2	ALPH2	"
11-15	F5.2	BETH1	"
16-20	F5.2	BETH2	"
1-50	10F5.2	TM(10)	Array containing the times (hr.) that output must be printed.

Appendix Table 2. (continued)

2. Initial conditions.
   The initial conditions, used for the example run, are listed in the function UIN (Z, NSOIL). Only pressure head values can be assigned to the initial conditions.

3. Soil properties.
   Analytical expressions for $\theta(h)$, $K(h)$, and $C(h)$ are defined in the functions FTH (U, NSOIL), FK (U, NSOIL), and FC (U, NSOIL), respectively.

4. Transient boundary conditions can be defined in the subroutine UNSTE (UTOP, UBOT, U1, V1, NZP1).

```
INITIALIZATIONS AND BOUNDARY CONDITIONS OF
         INFI-MODEL

NR. OF SPACESTEPS ..........   35
NR. OF TIMESTEPS ...........  600
TIMES OUTPUT IS PRINTED ...     2
SOIL TYPE .................     1
DEPTH OF PROFILE (CM) .....  70.00000
TOP BOUNDARY CONDITION .... -13.69000  ALPHA1=0.0  ALPHA2=1.0
BOTTOM BOUNDARY CONDITION . -61.50000  BETHA1=1.0  BETHA2=0.0
INITIAL TIMESTEP (HOURS) ..   0.00100
MODEL STOPS AT ............   6.00000 HOURS

OUTPUT IS PRINTED AT :
  1.00 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  HOURS
```

Note: Printout was specified for 1.00 hours. The remaining 9 options were not used

THE PHYSICAL PROPERTIES OF SOILNR 1

PRESSURE HEAD CM	THETA	HYDRAULIC CONDUCTIVITY CM/HR	WATERCAPACITY CM -1
-10.0	0.286	0.32481E+02	0.46993E-03
-20.0	0.270	0.15112E+02	0.31235E-02
-30.0	0.222	0.35635E+01	0.59319E-02
-40.0	0.164	0.98843E+00	0.51185E-02
-50.0	0.124	0.34987E+00	0.29881E-02
-60.0	0.102	0.14832E+00	0.15589E-02
-70.0	0.091	0.71588E-01	0.81842E-03
-80.0	0.084	0.38053E-01	0.44877E-03
-90.0	0.081	0.21784E-01	0.25877E-03
-100.0	0.079	0.13224E-01	0.15648E-03
-110.0	0.078	0.84180E-02	0.98712E-04
-120.0	0.077	0.55735E-02	0.64603E-04
-130.0	0.076	0.38140E-02	0.43655E-04
-140.0	0.076	0.26844E-02	0.30333E-04
-150.0	0.076	0.19356E-02	0.21595E-04
-160.0	0.076	0.14255E-02	0.15707E-04
-170.0	0.076	0.10695E-02	0.11643E-04
-180.0	0.075	0.81568E-03	0.87769E-05
-190.0	0.075	0.63128E-03	0.67172E-05
-200.0	0.075	0.49503E-03	0.52113E-05
-300.0	0.075	0.72438E-04	0.69886E-06
-400.0	0.075	0.18525E-04	0.16782E-06
-500.0	0.075	0.64329E-05	0.55489E-07
-600.0	0.075	0.27107E-05	0.22464E-07
-700.0	0.075	0.13054E-05	0.10458E-07
-800.0	0.075	0.69322E-06	0.53926E-08
-900.0	0.075	0.39665E-06	0.30067E-08
-1000.0	0.075	0.24072E-06	0.17829E-08
-1100.0	0.075	0.15322E-06	0.11113E-08
-1200.0	0.075	0.10144E-06	0.72174E-09
-1300.0	0.075	0.69411E-07	0.48526E-09
-1400.0	0.075	0.48851E-07	0.33599E-09
-1500.0	0.075	0.35225E-07	0.23863E-09
-2500.0	0.075	0.31281E-08	0.18938E-10
-3500.0	0.075	0.63480E-09	0.35690E-11
-4500.0	0.075	0.19288E-09	0.10261E-11
-5500.0	0.075	0.74508E-10	0.37925E-12
-6500.0	0.075	0.33753E-10	0.16561E-12
-7500.0	0.075	0.17129E-10	0.81436E-13
-8500.0	0.075	0.94639E-11	0.43774E-13
-9500.0	0.075	0.55861E-11	0.25213E-13
-10500.0	0.075	0.34760E-11	0.15347E-13
-11500.0	0.075	0.22585E-11	0.97739E-14
-12500.0	0.075	0.15211E-11	0.64634E-14
-13500.0	0.075	0.10562E-11	0.44123E-14
-14500.0	0.075	0.75273E-12	0.30956E-14
-15500.0	0.075	0.54872E-12	0.22237E-14

Note: Above tables were actually generated by the hydraulic relationships specified in the program.

VALUES FOR THETA,PRESSURE HEAD AND FLUX FOR THE SUCCESSIVE GRIDPOINTS AT TIME:
TOPBOUND.CONDITION -13.69000
BOTTOMBOUND.CONDITION -61.50000

GRIDPOINT	DEPTH(CM)	THETA	H(CM)	FLUX(CM/HR)
1	0.0	0.99851E-01	-0.61500E+02	-0.13200E+00
2	-0.20000E+01	0.99851E-01	-0.61500E+02	-0.13200E+00
3	-0.40000E+01	0.99851E-01	-0.61500E+02	-0.13200E+00
4	-0.60000E+01	0.99851E-01	-0.61500E+02	-0.13200E+00
5	-0.80000E+01	0.99851E-01	-0.61500E+02	-0.13200E+00
6	-0.10000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
7	-0.12000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
8	-0.14000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
9	-0.16000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
10	-0.18000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
11	-0.20000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
12	-0.22000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
13	-0.24000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
14	-0.26000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
15	-0.28000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
16	-0.30000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
17	-0.32000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
18	-0.34000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
19	-0.36000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
20	-0.38000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
21	-0.40000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
22	-0.42000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
23	-0.44000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
24	-0.46000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
25	-0.48000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
26	-0.50000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
27	-0.52000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
28	-0.54000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
29	-0.56000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
30	-0.58000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
31	-0.60000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
32	-0.62000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
33	-0.64000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
34	-0.66000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
35	-0.68000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00
36	-0.70000E+02	0.99851E-01	-0.61500E+02	-0.13200E+00

WATER IN PROFILE AT TIME 0.0 : 6.98956 CM

Note: Above data were generated by assigning values to the parameters that specify the initial and boundary conditions.

Appendix Table 3. Description and Listing of Output

The following variables are printed for the initial time (TIM=0) and for the selected times [TM(ITIM)].

TIM	- Time since start of simulation (hr.).
KOUNT	- Time step number.
UTOP	- Top boundary condition - pressure head or flux.
UBOT	- Bottom boundary condition - pressure head or flux.
SAVDEN	- Increase in stored moisture over current time step as calculated from two consecutive water content profiles (cm).
SAVMAS	- Moisture intake by profile over current timestep as calculated from fluxes at the boundaries (cm).
TM(ITIM+1)	- Next time that output will be printed (hr.).
Z	- Depths of all gridpoints at time TM(ITIM).
TM	- Theta's of all gridpoints at time TM(ITIM).
U2	- Pressure heads of all gridpoints at time TM(ITIM).
V2	- Fluxes of all gridpoints at time TM(ITIM).

The following variables are listed at all time steps:

TIM	- Time since start of simulation (hr.).
DT	- Time step size (hr.).
XMB	- Absolute mass balance in current time step (cm).
XMB1	- Relative mass balance in current time step (pct.).
RMAXK	- The maximum difference in pressure head and flux over all gridpoints, which occurred between the first and last iteration of the current time step (cm or cm/hr.).
IT	- Number of iterations which were needed to solve the linear system in current time step.

TIME HRS	TIMESTEP HRS	ABS.MASSBAL. CM	REL.MASSBAL. PERC	ERRORCONTROL CM,CM/HR	NR. OF ITERATI
0.0	0.10000E-02	0.69587E-01	0.10265E+04	0.31667E+01	16
0.0	0.50000E-03	0.25821E-02	0.76180E+02	0.48493E+01	11
0.0	0.25000E-03	0.11649E-02	0.68732E+02	0.30858E+01	10
0.0	0.12500E-03	0.62680E-03	0.73971E+02	0.17831E+01	8
0.00012	0.12500E-03	0.11591E-03	0.68395E+01	0.73695E+00	5
0.00025	0.12500E-03	0.81167E-04	0.47894E+01	0.44444E+00	5
0.00037	0.25000E-03	0.25140E-03	0.74310E+01	0.74705E+00	12
0.00062	0.25000E-03	0.18382E-03	0.54633E+01	0.48906E+00	6
0.00087	0.25000E-03	0.14695E-03	0.44066E+01	0.43755E+00	5
0.00112	0.25000E-03	0.79749E-04	0.24207E+01	0.41311E+00	5
0.00137	0.50000E-03	0.29158E-03	0.43769E+01	0.75484E+00	6
0.00187	0.50000E-03	0.37909E-03	0.55919E+01	0.56713E+00	7
0.00237	0.50000E-03	0.24115E-03	0.35572E+01	0.55078E+00	6
0.00287	0.50000E-03	0.11287E-03	0.16650E+01	0.52379E+00	6
0.00337	0.50000E-03	0.74804E-05	0.11034E+00	0.49387E+00	5
0.00387	0.10000E-02	0.74792E-03	0.55160E+01	0.62760E+00	6
0.00487	0.10000E-02	0.64250E-03	0.47381E+01	0.60754E+00	6
0.00587	0.10000E-02	0.57108E-03	0.42112E+01	0.57739E+00	5
0.00687	0.10000E-02	0.50421E-03	0.37181E+01	0.54030E+00	5
0.00787	0.10000E-02	0.45870E-03	0.33824E+01	0.51410E+00	5
0.00887	0.10000E-02	0.42545E-03	0.31373E+01	0.48870E+00	5
0.00987	0.10000E-02	0.38732E-03	0.28560E+01	0.46665E+00	5
0.01087	0.10000E-02	0.36040E-03	0.26575E+01	0.44625E+00	5
0.01187	0.10000E-02	0.33114E-03	0.24418E+01	0.42894E+00	5
0.01287	0.10000E-02	0.31126E-03	0.22952E+01	0.41223E+00	5
0.01387	0.10000E-02	0.28828E-03	0.21257E+01	0.39820E+00	5
0.01487	0.10000E-02	0.27534E-03	0.20303E+01	0.38429E+00	5
0.01587	0.10000E-02	0.25063E-03	0.18481E+01	0.37269E+00	5
0.01687	0.10000E-02	0.23357E-03	0.17223E+01	0.36085E+00	5
0.01787	0.10000E-02	0.21940E-03	0.16178E+01	0.35112E+00	5
0.01887	0.10000E-02	0.20510E-03	0.15124E+01	0.34092E+00	5
0.01987	0.10000E-02	0.18585E-03	0.13704E+01	0.33265E+00	5
0.02087	0.10000E-02	0.17295E-03	0.12753E+01	0.32409E+00	5
0.02187	0.10000E-02	0.15913E-03	0.11734E+01	0.31676E+00	5
0.02287	0.10000E-02	0.14923E-03	0.11004E+01	0.30937E+00	5
0.02387	0.10000E-02	0.12572E-03	0.92705E+00	0.30285E+00	5
0.02487	0.10000E-02	0.11659E-03	0.85970E+00	0.29652E+00	5
0.02587	0.10000E-02	0.10199E-03	0.75210E+00	0.29061E+00	5
0.02687	0.10000E-02	0.94935E-04	0.70005E+00	0.28518E+00	5
0.02787	0.20000E-02	0.70370E-03	0.25945E+01	0.50775E+00	5
0.02987	0.20000E-02	0.67665E-03	0.24950E+01	0.49091E+00	5
0.03187	0.20000E-02	0.64457E-03	0.23770E+01	0.47857E+00	5
0.03387	0.20000E-02	0.61528E-03	0.22690E+01	0.46543E+00	5
0.03587	0.20000E-02	0.58705E-03	0.21649E+01	0.45434E+00	5
0.03787	0.20000E-02	0.54123E-03	0.19959E+01	0.44389E+00	5
0.03987	0.20000E-02	0.49560E-03	0.18276E+01	0.43425E+00	5
0.04187	0.20000E-02	0.43162E-03	0.15917E+01	0.42253E+00	5
0.04387	0.20000E-02	0.35331E-03	0.13029E+01	0.41684E+00	5
0.04587	0.20000E-02	0.24776E-03	0.91370E+00	0.40879E+00	5
0.04787	0.20000E-02	0.11775E-03	0.43426E+00	0.40063E+00	5
0.04987	0.20000E-02	0.55466E-04	0.20456E+00	0.39482E+00	5
0.05187	0.40000E-02	0.18256E-02	0.33665E+01	0.69714E+00	6
0.05187	0.20000E-02	0.39461E-03	0.14556E+01	0.38684E+00	5
0.05387	0.20000E-02	0.43209E-03	0.15942E+01	0.38158E+00	5
0.05587	0.20000E-02	0.44197E-03	0.16306E+01	0.37655E+00	5
0.05787	0.20000E-02	0.46328E-03	0.17091E+01	0.36994E+00	5
0.05987	0.20000E-02	0.46558E-03	0.17175E+01	0.36576E+00	5
0.06187	0.20000E-02	0.46638E-03	0.17204E+01	0.36078E+00	5

0.06387	0.20000E-02	0.45802E-03	0.16894E+01	0.35628E+00	5
0.06587	0.20000E-02	0.44167E-03	0.16291E+01	0.35283E+00	5
0.06787	0.20000E-02	0.42223E-03	0.15574E+01	0.34820E+00	5
0.06987	0.20000E-02	0.39205E-03	0.14460E+01	0.34632E+00	5
0.07187	0.20000E-02	0.36702E-03	0.13537E+01	0.34266E+00	5
0.07387	0.20000E-02	0.32202E-03	0.11877E+01	0.33754E+00	5
0.07587	0.20000E-02	0.28778E-03	0.10614E+01	0.33130E+00	5
0.07787	0.20000E-02	0.25456E-03	0.93886E+00	0.32635E+00	5
0.07987	0.20000E-02	0.21188E-03	0.78143E+00	0.32665E+00	5
0.08187	0.20000E-02	0.18166E-03	0.66997E+00	0.32525E+00	5
0.08387	0.20000E-02	0.14586E-03	0.53792E+00	0.32242E+00	5
0.08587	0.20000E-02	0.11852E-03	0.43708E+00	0.31843E+00	5
0.08787	0.20000E-02	0.10766E-03	0.39705E+00	0.31693E+00	5
0.08987	0.20000E-02	0.89295E-04	0.32931E+00	0.31413E+00	5
0.09187	0.40000E-02	0.12306E-02	0.22692E+01	0.57155E+00	6
0.09187	0.20000E-02	0.27170E-03	0.10020E+01	0.30849E+00	5
0.09387	0.20000E-02	0.28569E-03	0.10535E+01	0.30697E+00	5
0.09587	0.20000E-02	0.29276E-03	0.10797E+01	0.30546E+00	5
0.09787	0.20000E-02	0.30178E-03	0.11130E+01	0.30254E+00	5
0.09987	0.20000E-02	0.31298E-03	0.11543E+01	0.30141E+00	5
0.10187	0.20000E-02	0.31953E-03	0.11785E+01	0.29905E+00	5
0.10387	0.20000E-02	0.32685E-03	0.12055E+01	0.29758E+00	5
0.10587	0.20000E-02	0.32832E-03	0.12109E+01	0.29587E+00	5
0.10787	0.20000E-02	0.34172E-03	0.12604E+01	0.29376E+00	5
0.10987	0.20000E-02	0.34418E-03	0.12695E+01	0.29298E+00	5
0.11187	0.20000E-02	0.33003E-03	0.12173E+01	0.29103E+00	5
0.11387	0.20000E-02	0.33325E-03	0.12291E+01	0.29003E+00	5
0.11587	0.20000E-02	0.32920E-03	0.12142E+01	0.28914E+00	5
0.11787	0.20000E-02	0.31394E-03	0.11580E+01	0.28717E+00	5
0.11987	0.20000E-02	0.31001E-03	0.11435E+01	0.28706E+00	5
0.12187	0.20000E-02	0.29520E-03	0.10888E+01	0.28607E+00	5
0.12387	0.20000E-02	0.28401E-03	0.10476E+01	0.28406E+00	5
0.12587	0.20000E-02	0.26999E-03	0.99584E+00	0.28116E+00	5
0.12787	0.20000E-02	0.25684E-03	0.94735E+00	0.28006E+00	5
0.12987	0.20000E-02	0.23850E-03	0.87970E+00	0.28056E+00	5
0.13187	0.20000E-02	0.21951E-03	0.80966E+00	0.27993E+00	5
0.13387	0.20000E-02	0.20507E-03	0.75640E+00	0.27833E+00	5
0.13587	0.20000E-02	0.18483E-03	0.68174E+00	0.27736E+00	5
0.13787	0.20000E-02	0.16503E-03	0.60870E+00	0.27648E+00	5
0.13987	0.20000E-02	0.13909E-03	0.51303E+00	0.27535E+00	4
0.14187	0.20000E-02	0.12158E-03	0.44845E+00	0.27461E+00	4
0.14387	0.20000E-02	0.10374E-03	0.38264E+00	0.27348E+00	4
0.14587	0.20000E-02	0.73902E-04	0.27258E+00	0.27276E+00	4
0.14787	0.40000E-02	0.10269E-02	0.18933E+01	0.50437E+00	6
0.14787	0.20000E-02	0.22977E-03	0.84741E+00	0.26937E+00	4
0.14987	0.20000E-02	0.24364E-03	0.89849E+00	0.26883E+00	4
0.15187	0.20000E-02	0.24758E-03	0.91303E+00	0.26856E+00	4
0.15387	0.20000E-02	0.25582E-03	0.94346E+00	0.26742E+00	4
0.15587	0.20000E-02	0.26355E-03	0.97197E+00	0.26565E+00	4
0.15787	0.20000E-02	0.26674E-03	0.98372E+00	0.26577E+00	4
0.15987	0.20000E-02	0.27001E-03	0.99578E+00	0.26501E+00	4
0.16187	0.20000E-02	0.27936E-03	0.10302E+01	0.26349E+00	4
0.16387	0.20000E-02	0.27621E-03	0.10186E+01	0.26351E+00	4
0.16587	0.20000E-02	0.27734E-03	0.10228E+01	0.26313E+00	4
0.16787	0.20000E-02	0.27991E-03	0.10323E+01	0.26184E+00	4
0.16987	0.20000E-02	0.27085E-03	0.99888E+00	0.26163E+00	4
0.17187	0.20000E-02	0.27150E-03	0.10013E+01	0.26175E+00	4
0.17387	0.20000E-02	0.26441E-03	0.97512E+00	0.26098E+00	4
0.17587	0.20000E-02	0.25833E-03	0.95270E+00	0.25939E+00	4
0.17787	0.20000E-02	0.24711E-03	0.91132E+00	0.25834E+00	4

0.17987	0.20000E-02	0.23522E-03	0.86746E+00	0.25932E+00	4
0.18187	0.20000E-02	0.22954E-03	0.84652E+00	0.25941E+00	4
0.18387	0.20000E-02	0.21241E-03	0.78334E+00	0.25859E+00	4
0.18587	0.20000E-02	0.20055E-03	0.73959E+00	0.25702E+00	4
0.18787	0.20000E-02	0.18655E-03	0.68798E+00	0.25589E+00	4
0.18987	0.20000E-02	0.16921E-03	0.62401E+00	0.25671E+00	4
0.19187	0.20000E-02	0.15352E-03	0.56617E+00	0.25668E+00	4
0.19387	0.20000E-02	0.13294E-03	0.49028E+00	0.25578E+00	4
0.19587	0.20000E-02	0.11471E-03	0.42305E+00	0.25493E+00	4
0.19787	0.20000E-02	0.91523E-04	0.33753E+00	0.25514E+00	4
0.19987	0.40000E-02	0.94771E-03	0.17476E+01	0.47503E+00	5
0.20387	0.40000E-02	0.95391E-03	0.17587E+01	0.47215E+00	5
0.20787	0.40000E-02	0.98097E-03	0.18081E+01	0.47106E+00	5
0.21187	0.40000E-02	0.98589E-03	0.18171E+01	0.46965E+00	5
0.21587	0.40000E-02	0.98956E-03	0.18239E+01	0.46732E+00	5
0.21987	0.40000E-02	0.97266E-03	0.17929E+01	0.46676E+00	5
0.22387	0.40000E-02	0.94804E-03	0.17476E+01	0.46599E+00	5
0.22787	0.40000E-02	0.91302E-03	0.16832E+01	0.46430E+00	5
0.23187	0.40000E-02	0.88808E-03	0.16372E+01	0.45941E+00	5
0.23587	0.40000E-02	0.83905E-03	0.15469E+01	0.46220E+00	5
0.23987	0.40000E-02	0.80216E-03	0.14789E+01	0.45975E+00	5
0.24387	0.40000E-02	0.75486E-03	0.13917E+01	0.45971E+00	5
0.24787	0.40000E-02	0.68444E-03	0.12618E+01	0.45824E+00	5
0.25187	0.40000E-02	0.60862E-03	0.11220E+01	0.45672E+00	5
0.25587	0.40000E-02	0.51257E-03	0.94491E+00	0.45640E+00	5
0.25987	0.40000E-02	0.38707E-03	0.71353E+00	0.45578E+00	5
0.26387	0.40000E-02	0.22575E-03	0.41613E+00	0.45501E+00	5
0.26787	0.40000E-02	0.53942E-04	0.99430E-01	0.45417E+00	5
0.27187	0.80000E-02	0.34174E-02	0.31497E+01	0.81332E+00	7
0.27187	0.40000E-02	0.75498E-03	0.13919E+01	0.45023E+00	5
0.27587	0.40000E-02	0.77471E-03	0.14285E+01	0.44788E+00	5
0.27987	0.40000E-02	0.82779E-03	0.15264E+01	0.44767E+00	5
0.28387	0.40000E-02	0.86918E-03	0.16027E+01	0.44526E+00	5
0.28787	0.40000E-02	0.90736E-03	0.16731E+01	0.44499E+00	5
0.29187	0.40000E-02	0.92894E-03	0.17129E+01	0.44424E+00	5
0.29587	0.40000E-02	0.94143E-03	0.17359E+01	0.44378E+00	5
0.29987	0.40000E-02	0.93696E-03	0.17277E+01	0.44465E+00	5
0.30387	0.40000E-02	0.92414E-03	0.17040E+01	0.44414E+00	
0.30787	0.40000E-02	0.90379E-03	0.16665E+01	0.43938E+00	
0.31187	0.40000E-02	0.86737E-03	0.15994E+01	0.44249E+00	
0.31587	0.40000E-02	0.83131E-03	0.15329E+01	0.44390E+00	
0.31987	0.40000E-02	0.78544E-03	0.14483E+01	0.44081E+00	
0.32387	0.40000E-02	0.74306E-03	0.13702E+01	0.44287E+00	
0.32787	0.40000E-02	0.69350E-03	0.12788E+01	0.44201E+00	
0.33187	0.40000E-02	0.64018E-03	0.11805E+01	0.44197E+00	
0.33587	0.40000E-02	0.58904E-03	0.10862E+01	0.44166E+00	
0.33987	0.40000E-02	0.52881E-03	0.97510E+00	0.44182E+00	
0.34387	0.40000E-02	0.46989E-03	0.86646E+00	0.44071E+00	
0.34787	0.40000E-02	0.41091E-03	0.75770E+00	0.44135E+00	
0.35187	0.40000E-02	0.33954E-03	0.62609E+00	0.44159E+00	
0.35587	0.40000E-02	0.28417E-03	0.52399E+00	0.44105E+00	
0.35987	0.40000E-02	0.21815E-03	0.40226E+00	0.44104E+00	
0.36387	0.40000E-02	0.15283E-03	0.28180E+00	0.44158E+00	
0.36787	0.40000E-02	0.94622E-04	0.17448E+00	0.44075E+00	
0.37187	0.80000E-02	0.31750E-02	0.29267E+01	0.79088E+00	
0.37187	0.40000E-02	0.77865E-03	0.14358E+01	0.43410E+00	
0.37587	0.40000E-02	0.78881E-03	0.14546E+01	0.43421E+00	
0.37987	0.40000E-02	0.80255E-03	0.14799E+01	0.43226E+00	
0.38387	0.40000E-02	0.82129E-03	0.15145E+01	0.43318E+00	
0.38787	0.40000E-02	0.84102E-03	0.15509E+01	0.43156E+00	

0.39187	0.40000E-02	0.84940E-03	0.15664E+01	0.43290E+00	5
0.39587	0.40000E-02	0.85580E-03	0.15782E+01	0.43210E+00	5
0.39987	0.40000E-02	0.85303E-03	0.15730E+01	0.43317E+00	5
0.40387	0.40000E-02	0.84379E-03	0.15560E+01	0.43169E+00	5
0.40787	0.40000E-02	0.83157E-03	0.15334E+01	0.43393E+00	5
0.41187	0.40000E-02	0.81617E-03	0.15050E+01	0.43179E+00	5
0.41587	0.40000E-02	0.79113E-03	0.14588E+01	0.43389E+00	5
0.41987	0.40000E-02	0.76458E-03	0.14099E+01	0.43373E+00	5
0.42387	0.40000E-02	0.73716E-03	0.13593E+01	0.43321E+00	5
0.42787	0.40000E-02	0.71076E-03	0.13107E+01	0.43428E+00	5
0.43187	0.40000E-02	0.68176E-03	0.12572E+01	0.43346E+00	5
0.43587	0.40000E-02	0.64263E-03	0.11851E+01	0.43415E+00	5
0.43987	0.40000E-02	0.60433E-03	0.11145E+01	0.43435E+00	5
0.44387	0.40000E-02	0.54285E-03	0.10012E+01	0.43390E+00	5
0.44787	0.40000E-02	0.47502E-03	0.87610E+00	0.43411E+00	5
0.45187	0.40000E-02	0.38132E-03	0.70331E+00	0.43476E+00	5
0.45587	0.40000E-02	0.25252E-03	0.46574E+00	0.43467E+00	5
0.45987	0.40000E-02	0.10297E-03	0.18991E+00	0.43454E+00	5
0.46387	0.40000E-02	0.98765E-04	0.18214E+00	0.43530E+00	5
0.46787	0.80000E-02	0.32558E-02	0.30018E+01	0.78316E+00	7
0.46787	0.40000E-02	0.66519E-03	0.12266E+01	0.42786E+00	5
0.47187	0.40000E-02	0.70754E-03	0.13046E+01	0.42738E+00	5
0.47587	0.40000E-02	0.74533E-03	0.13741E+01	0.42898E+00	5
0.47987	0.40000E-02	0.78633E-03	0.14496E+01	0.42798E+00	5
0.48387	0.40000E-02	0.82985E-03	0.15298E+01	0.42856E+00	5
0.48787	0.40000E-02	0.86483E-03	0.15943E+01	0.42813E+00	5
0.49187	0.40000E-02	0.88668E-03	0.16345E+01	0.42809E+00	5
0.49587	0.40000E-02	0.89505E-03	0.16500E+01	0.42877E+00	5
0.49987	0.40000E-02	0.88584E-03	0.16330E+01	0.42856E+00	5
0.50387	0.40000E-02	0.87166E-03	0.16069E+01	0.43062E+00	5
0.50787	0.40000E-02	0.85053E-03	0.15680E+01	0.43018E+00	5
0.51187	0.40000E-02	0.82591E-03	0.15227E+01	0.42591E+00	5
0.51587	0.40000E-02	0.79766E-03	0.14707E+01	0.42778E+00	5
0.51987	0.40000E-02	0.77400E-03	0.14271E+01	0.43053E+00	5
0.52387	0.40000E-02	0.74142E-03	0.13671E+01	0.42887E+00	5
0.52787	0.40000E-02	0.70500E-03	0.12999E+01	0.43037E+00	5
0.53187	0.40000E-02	0.65243E-03	0.12030E+01	0.43038E+00	5
0.53587	0.40000E-02	0.58627E-03	0.10811E+01	0.42957E+00	5
0.53987	0.40000E-02	0.50324E-03	0.92796E+00	0.43094E+00	5
0.54387	0.40000E-02	0.39592E-03	0.73007E+00	0.43103E+00	5
0.54787	0.40000E-02	0.26035E-03	0.48007E+00	0.43120E+00	5
0.55187	0.40000E-02	0.10329E-03	0.19046E+00	0.43159E+00	5
0.55587	0.40000E-02	0.81509E-04	0.15029E+00	0.43209E+00	5
0.55987	0.80000E-02	0.31750E-02	0.29275E+01	0.77877E+00	7
0.55987	0.40000E-02	0.75164E-03	0.13860E+01	0.42856E+00	5
0.56387	0.40000E-02	0.74208E-03	0.13685E+01	0.42824E+00	5
0.56787	0.40000E-02	0.76240E-03	0.14061E+01	0.42600E+00	5
0.57187	0.40000E-02	0.78604E-03	0.14499E+01	0.42644E+00	5
0.57587	0.40000E-02	0.81030E-03	0.14950E+01	0.42579E+00	5
0.57987	0.40000E-02	0.83086E-03	0.15333E+01	0.42533E+00	5
0.58387	0.40000E-02	0.83852E-03	0.15478E+01	0.42595E+00	5
0.58787	0.40000E-02	0.84198E-03	0.15545E+01	0.42465E+00	5
0.59187	0.40000E-02	0.83756E-03	0.15468E+01	0.42657E+00	5
0.59587	0.40000E-02	0.81438E-03	0.15044E+01	0.42504E+00	5
0.59987	0.40000E-02	0.78756E-03	0.14554E+01	0.42767E+00	5
0.60387	0.40000E-02	0.74297E-03	0.13736E+01	0.42647E+00	5
0.60787	0.40000E-02	0.67884E-03	0.12557E+01	0.42481E+00	5
0.61187	0.40000E-02	0.59083E-03	0.10936E+01	0.42792E+00	5
0.61587	0.40000E-02	0.46334E-03	0.85830E+00	0.42681E+00	5
0.61987	0.40000E-02	0.28706E-03	0.53226E+00	0.42822E+00	5

0.62387	0.40000E-02	0.30607E-04	0.56816E-01	0.42878E+00	6
0.62787	0.80000E-02	0.27059E-02	0.25016E+01	0.77220E+00	7
0.62787	0.40000E-02	0.47532E-03	0.87882E+00	0.42264E+00	5
0.63187	0.40000E-02	0.72610E-03	0.13360E+01	0.42417E+00	5
0.63587	0.40000E-02	0.75778E-03	0.13937E+01	0.42227E+00	5
0.63987	0.40000E-02	0.78231E-03	0.14385E+01	0.42320E+00	5
0.64387	0.40000E-02	0.80040E-03	0.14716E+01	0.42225E+00	5
0.64787	0.40000E-02	0.81253E-03	0.14939E+01	0.42232E+00	5
0.65187	0.40000E-02	0.80863E-03	0.14868E+01	0.42261E+00	5
0.65587	0.40000E-02	0.79647E-03	0.14646E+01	0.42190E+00	5
0.65987	0.40000E-02	0.76655E-03	0.14097E+01	0.42334E+00	5
0.66387	0.40000E-02	0.72026E-03	0.13247E+01	0.42234E+00	5
0.66787	0.40000E-02	0.66161E-03	0.12169E+01	0.42134E+00	5
0.67187	0.40000E-02	0.59110E-03	0.10873E+01	0.42416E+00	5
0.67587	0.40000E-02	0.50682E-03	0.93230E+00	0.42279E+00	5
0.67987	0.40000E-02	0.41860E-03	0.77005E+00	0.42239E+00	5
0.68387	0.40000E-02	0.31868E-03	0.58622E+00	0.42439E+00	5
0.68787	0.40000E-02	0.21744E-03	0.39997E+00	0.42384E+00	5
0.69187	0.40000E-02	0.12740E-03	0.23433E+00	0.42464E+00	5
0.69587	0.40000E-02	0.46998E-04	0.86428E-01	0.42540E+00	5
0.69987	0.80000E-02	0.32222E-02	0.29666E+01	0.76554E+00	7
0.69987	0.40000E-02	0.73612E-03	0.13552E+01	0.41783E+00	5
0.70387	0.40000E-02	0.69833E-03	0.12871E+01	0.41953E+00	5
0.70787	0.40000E-02	0.72971E-03	0.13446E+01	0.41853E+00	5
0.71187	0.40000E-02	0.76598E-03	0.14111E+01	0.41998E+00	5
0.71587	0.40000E-02	0.79966E-03	0.14729E+01	0.41989E+00	5
0.71987	0.40000E-02	0.82859E-03	0.15259E+01	0.42003E+00	5
0.72387	0.40000E-02	0.84677E-03	0.15591E+01	0.42119E+00	5
0.72787	0.40000E-02	0.85336E-03	0.15709E+01	0.42028E+00	5
0.73187	0.40000E-02	0.85106E-03	0.15664E+01	0.42268E+00	5
0.73587	0.40000E-02	0.83220E-03	0.15314E+01	0.42303E+00	5
0.73987	0.40000E-02	0.80183E-03	0.14752E+01	0.42441E+00	5
0.74387	0.40000E-02	0.75811E-03	0.13945E+01	0.42173E+00	5
0.74787	0.40000E-02	0.69761E-03	0.12830E+01	0.42417E+00	5
0.75187	0.40000E-02	0.62400E-03	0.11475E+01	0.42577E+00	5
0.75587	0.40000E-02	0.51850E-03	0.95346E+00	0.42334E+00	5
0.75987	0.40000E-02	0.38943E-03	0.71624E+00	0.42625E+00	5
0.76387	0.40000E-02	0.21428E-03	0.39431E+00	0.42553E+00	5
0.76787	0.40000E-02	0.17554E-04	0.32334E-01	0.42666E+00	5
0.77187	0.80000E-02	0.46932E-02	0.43910E+01	0.77115E+00	7
0.77187	0.40000E-02	0.14119E-02	0.26362E+01	0.42425E+00	5
0.77187	0.20000E-02	0.47335E-03	0.17662E+01	0.22417E+00	4
0.77387	0.20000E-02	0.17353E-03	0.65591E+00	0.22505E+00	4
0.77587	0.20000E-02	0.17538E-03	0.66379E+00	0.22502E+00	4
0.77787	0.20000E-02	0.18104E-03	0.68616E+00	0.22534E+00	4
0.77987	0.20000E-02	0.18748E-03	0.71170E+00	0.22530E+00	4
0.78187	0.20000E-02	0.19451E-03	0.73986E+00	0.22474E+00	4
0.78387	0.20000E-02	0.19936E-03	0.76005E+00	0.22509E+00	4
0.78587	0.20000E-02	0.20185E-03	0.77171E+00	0.22481E+00	4
0.78787	0.20000E-02	0.20211E-03	0.77516E+00	0.22468E+00	4
0.78987	0.20000E-02	0.20194E-03	0.77740E+00	0.22482E+00	4
0.79187	0.20000E-02	0.19908E-03	0.76963E+00	0.22442E+00	4
0.79387	0.20000E-02	0.19229E-03	0.74699E+00	0.22485E+00	4
0.79587	0.20000E-02	0.18489E-03	0.72221E+00	0.22491E+00	4
0.79787	0.20000E-02	0.17587E-03	0.69123E+00	0.22467E+00	4
0.79987	0.20000E-02	0.16635E-03	0.65840E+00	0.22548E+00	4
0.80187	0.20000E-02	0.15891E-03	0.63395E+00	0.22563E+00	4
0.80387	0.20000E-02	0.14921E-03	0.60051E+00	0.22514E+00	4
0.80587	0.20000E-02	0.13838E-03	0.56242E+00	0.22413E+00	4
0.80787	0.20000E-02	0.12850E-03	0.52805E+00	0.22254E+00	4

0.80987	0.20000E-02	0.12180E-03	0.50666E+00	0.22345E+00	4
0.81187	0.20000E-02	0.11650E-03	0.49122E+00	0.22452E+00	4
0.81387	0.20000E-02	0.11218E-03	0.48011E+00	0.22483E+00	4
0.81587	0.20000E-02	0.11200E-03	0.48734E+00	0.22445E+00	4
0.81787	0.20000E-02	0.11407E-03	0.50539E+00	0.22338E+00	4
0.81987	0.20000E-02	0.11510E-03	0.52015E+00	0.22287E+00	4
0.82187	0.20000E-02	0.11709E-03	0.54072E+00	0.22263E+00	4
0.82387	0.20000E-02	0.12626E-03	0.59693E+00	0.22164E+00	4
0.82587	0.20000E-02	0.13220E-03	0.64114E+00	0.22042E+00	5
0.82787	0.20000E-02	0.13583E-03	0.67711E+00	0.21934E+00	5
0.82986	0.20000E-02	0.14465E-03	0.74266E+00	0.21748E+00	5
0.83186	0.20000E-02	0.14897E-03	0.78944E+00	0.21593E+00	5
0.83386	0.20000E-02	0.15606E-03	0.85545E+00	0.21372E+00	5
0.83586	0.20000E-02	0.15564E-03	0.88431E+00	0.21134E+00	5
0.83786	0.20000E-02	0.16136E-03	0.95227E+00	0.20867E+00	5
0.83986	0.20000E-02	0.16119E-03	0.99015E+00	0.20529E+00	5
0.84186	0.20000E-02	0.16366E-03	0.10485E+01	0.20228E+00	5
0.84386	0.20000E-02	0.15778E-03	0.10564E+01	0.19851E+00	5
0.84586	0.20000E-02	0.15636E-03	0.10961E+01	0.19405E+00	5
0.84786	0.20000E-02	0.15284E-03	0.11237E+01	0.18937E+00	5
0.84986	0.20000E-02	0.14869E-03	0.11486E+01	0.18486E+00	5
0.85186	0.20000E-02	0.14285E-03	0.11613E+01	0.17971E+00	5
0.85386	0.20000E-02	0.13689E-03	0.11729E+01	0.17401E+00	5
0.85586	0.20000E-02	0.13408E-03	0.12124E+01	0.16792E+00	5
0.85786	0.20000E-02	0.12489E-03	0.11934E+01	0.16147E+00	5
0.85986	0.20000E-02	0.12240E-03	0.12374E+01	0.15479E+00	5
0.86186	0.20000E-02	0.11223E-03	0.12017E+01	0.14799E+00	5
0.86386	0.20000E-02	0.11125E-03	0.12628E+01	0.14114E+00	5
0.86586	0.20000E-02	0.10242E-03	0.12335E+01	0.13428E+00	5
0.86786	0.20000E-02	0.98299E-04	0.12572E+01	0.12801E+00	4
0.86986	0.40000E-02	0.80067E-03	0.54243E+01	0.89046E+00	9
0.87386	0.40000E-02	0.19604E-02	0.15827E+02	0.21053E+00	7
0.87386	0.20000E-02	0.72219E-03	0.11189E+02	0.14962E+00	5
0.87586	0.20000E-02	0.28232E-03	0.51928E+01	0.82170E-01	5
0.87786	0.20000E-02	0.17637E-03	0.35601E+01	0.69724E-01	4
0.87986	0.20000E-02	0.14181E-03	0.30732E+01	0.64670E-01	4
0.88186	0.20000E-02	0.12559E-03	0.29013E+01	0.60809E-01	4
0.88386	0.20000E-02	0.11451E-03	0.28133E+01	0.57330E-01	4
0.88586	0.20000E-02	0.10581E-03	0.27626E+01	0.54037E-01	4
0.88786	0.20000E-02	0.10080E-03	0.27958E+01	0.50858E-01	4
0.88986	0.20000E-02	0.95442E-04	0.28121E+01	0.47859E-01	4
0.89186	0.40000E-02	0.25425E-03	0.40291E+01	0.78990E-01	5
0.89586	0.40000E-02	0.30736E-03	0.53764E+01	0.70214E-01	5
0.89986	0.40000E-02	0.27216E-03	0.53263E+01	0.62480E-01	5
0.90386	0.40000E-02	0.24154E-03	0.52863E+01	0.55518E-01	5
0.90786	0.40000E-02	0.22688E-03	0.55595E+01	0.50367E-01	4
0.91186	0.40000E-02	0.19110E-03	0.52384E+01	0.44788E-01	4
0.91586	0.40000E-02	0.16360E-03	0.49991E+01	0.39921E-01	4
0.91986	0.40000E-02	0.15280E-03	0.51986E+01	0.35593E-01	4
0.92386	0.40000E-02	0.13357E-03	0.50546E+01	0.31733E-01	4
0.92786	0.40000E-02	0.11809E-03	0.49660E+01	0.28337E-01	4
0.93186	0.40000E-02	0.10475E-03	0.48901E+01	0.25326E-01	4
0.93586	0.40000E-02	0.93143E-04	0.48222E+01	0.22631E-01	4
0.93986	0.80000E-02	0.11249E-03	0.31813E+01	0.34768E-01	4
0.94786	0.80000E-02	0.24709E-03	0.78451E+01	0.29356E-01	4
0.95586	0.80000E-02	0.20236E-03	0.75563E+01	0.25050E-01	4
0.96386	0.80000E-02	0.17216E-03	0.75469E+01	0.21329E-01	4
0.97186	0.80000E-02	0.14501E-03	0.74589E+01	0.18155E-01	4
0.97986	0.80000E-02	0.12391E-03	0.74707E+01	0.15414E-01	4
0.98786	0.80000E-02	0.10360E-03	0.73182E+01	0.13129E-01	4

VALUES FOR THETA,PRESSURE HEAD AND FLUX FOR THE SUCCESSIVE GRIDPOINTS AT TIME:1.0
TOPBOUND.CONDITION -13.69000
BOTTOMBOUND.CONDITION -61.50000

GRIDPOINT	DEPTH(CM)	THETA	H(CM)	FLUX(CM/HR)
1	0.0	0.26747E+00	-0.20726E+02	-0.13690E+02
2	-0.35197E+02	0.26722E+00	-0.20799E+02	-0.13666E+02
3	-0.45452E+02	0.26658E+00	-0.20987E+02	-0.13649E+02
4	-0.51266E+02	0.26550E+00	-0.21290E+02	-0.13635E+02
5	-0.55202E+02	0.26399E+00	-0.21703E+02	-0.13622E+02
6	-0.58086E+02	0.26201E+00	-0.22220E+02	-0.13611E+02
7	-0.60289E+02	0.25953E+00	-0.22833E+02	-0.13601E+02
8	-0.62017E+02	0.25654E+00	-0.23533E+02	-0.13592E+02
9	-0.63395E+02	0.25302E+00	-0.24313E+02	-0.13584E+02
10	-0.64507E+02	0.24894E+00	-0.25165E+02	-0.13577E+02
11	-0.65412E+02	0.24431E+00	-0.26082E+02	-0.13570E+02
12	-0.66155E+02	0.23914E+00	-0.27057E+02	-0.13564E+02
13	-0.66767E+02	0.23344E+00	-0.28087E+02	-0.13558E+02
14	-0.67275E+02	0.22725E+00	-0.29165E+02	-0.13553E+02
15	-0.67698E+02	0.22062E+00	-0.30289E+02	-0.13549E+02
16	-0.68052E+02	0.21362E+00	-0.31455E+02	-0.13546E+02
17	-0.68349E+02	0.20632E+00	-0.32661E+02	-0.13543E+02
18	-0.68599E+02	0.19881E+00	-0.33904E+02	-0.13540E+02
19	-0.68811E+02	0.19118E+00	-0.35182E+02	-0.13538E+02
20	-0.68991E+02	0.18353E+00	-0.36494E+02	-0.13536E+02
21	-0.69145E+02	0.17593E+00	-0.37839E+02	-0.13535E+02
22	-0.69275E+02	0.16849E+00	-0.39215E+02	-0.13534E+02
23	-0.69388E+02	0.16126E+00	-0.40623E+02	-0.13533E+02
24	-0.69484E+02	0.15432E+00	-0.42060E+02	-0.13533E+02
25	-0.69567E+02	0.14771E+00	-0.43526E+02	-0.13532E+02
26	-0.69638E+02	0.14145E+00	-0.45022E+02	-0.13532E+02
27	-0.69700E+02	0.13559E+00	-0.46546E+02	-0.13532E+02
28	-0.69754E+02	0.13013E+00	-0.48098E+02	-0.13532E+02
29	-0.69801E+02	0.12507E+00	-0.49678E+02	-0.13532E+02
30	-0.69842E+02	0.12041E+00	-0.51286E+02	-0.13532E+02
31	-0.69877E+02	0.11613E+00	-0.52921E+02	-0.13532E+02
32	-0.69909E+02	0.11223E+00	-0.54583E+02	-0.13532E+02
33	-0.69936E+02	0.10867E+00	-0.56271E+02	-0.13532E+02
34	-0.69960E+02	0.10543E+00	-0.57983E+02	-0.13532E+02
35	-0.69981E+02	0.10250E+00	-0.59730E+02	-0.13532E+02
36	-0.70000E+02	0.99851E-01	-0.61500E+02	-0.13532E+02

CHANGE IN STORAGE IN THE WHOLE PROFILE 0.114227E+02 CM
TOTAL FLUX DIFFERENCE BETWEEN TOP AND BOTTOM TILL NOW 0.115647E+02 CM

COMPUTATIONS WILL PROCEED TILL 6.0 HOURS

```
C   Appendix Table 4.  Listing of WAFLOW
C ****************************************************************
C * W A F L O W                                                  *
C *                A ONE DIMENSIONAL SIMULATION MODEL              *
C *                                                              *
C *                                       VERSION MARCH 1982     *
C ****************************************************************
C
C




      COMMON IFLOW,NSOIL,TIM
      DIMENSION U0(150),U1(150),U2(150),V0(150),V1(150)
      DIMENSION V2(310),A(310,5),RH(310),A1(310,7),XL(2100)
      DIMENSION US(150),VS(150),TH(150),Z(150),TM(10)
      INTEGER FL /'AAAA'/,J1/'EVAP'/,J2/'INFI'/
C
C
C  ***** THE DATA ARE READ FROM DISK ****************************
      READ(3,25) IFLOW,NZ,NT,NTOUT,NSOIL
      READ(3,30) ZBOT,UTOP,UBOT,DT,TEND,ALPH1,ALPH2,BETH1,BETH2,
     1(TM(I),I=1,NTOUT)
25    FORMAT(5I5)
30    FORMAT(5F10.5,/,4F5.2,/,10F5.2)
      IF(IFLOW.EQ.0) FL=J1
      IF(IFLOW.EQ.1) FL=J2
C
C
C ****** THESE DATA ARE WRITTEN TO UNIT 6 ************************
40    WRITE(6,45) FL,NZ,NT,NTOUT,NSOIL,ZBOT,UTOP,ALPH1,ALPH2,UBOT,
     1BETH1,BETH2,DT,TEND,TM
45    FORMAT(' INITIALIZATIONS AND BOUNDARY CONDITIONS OF',2X,/,
     110X,A4,'-MODEL',3(/),
     2' NR. OF SPACESTEPS .........',I5,/,
     3' NR. OF TIMESTEPS ..........',I5,/,
     4' TIMES OUTPUT IS PRINTED ...',I5,/,
     5' SOIL TYPE .................',I5,/,
     6' DEPTH OF PROFILE (CM) .....',F10.5,/,
     7' TOP BOUNDARY CONDITION ....',F10.5,'  ALPHA1=',F3.1,'  ALPHA2=',
     8F3.1,/,
     8' BOTTOM BOUNDARY CONDITION .',F10.5,'  BETHA1=',F3.1,'  BETHA2=',
     9F3.1,/,
     9' INITIAL TIMESTEP (HOURS) ..',F10.5,/,
     1' MODEL STOPS AT ............',F10.5,' HOURS',2(/),
     2' OUTPUT IS PRINTED AT : ',/,2X,10F5.2,' HOURS',3(/))
C
```

```
C
C ***************************************************************
C IFLOW=0 ............ EVAPORATION MODEL                          *
C IFLOW=1 ............ INFILTRATION MODEL                         *
C NSOIL=1 ............ HAVERKAMP MODEL                            *
C NSOIL=2 ............ KLUTE MODEL                                *
C NSOIL=3 ............ DANE MODEL
C                                                                 *
C TOP BOUNDARY CONDITION:                                         *
C              CONSTANT FLUX ---> ALPHA1=0 AND ALPHA2=1           *
C    CONSTANT PRESSURE HEAD ---> ALPHA1=1 AND ALPHA2=0           *
C BOTTOM BOUNDARY CONDITION:                                      *
C              CONSTANT FLUX ---> BETHA1=0 AND BETHA2=1           *
C    CONSTANT PRESSURE HEAD ---> BETHA1=1 AND BETHA2=0           *
C                                                                 *
C                                                                 *
C  R E M E M B E R   T H E   D A R C Y   C O N V E N T I O N     *
C                                                                 *
C         POSITIVE FLUX ----> UPWARD FLOW                         *
C         NEGATIVE FLUX ----> DOWNWARD FLOW                       *
C                                                                 *
C ***************************************************************
C
C
C
C
C       LISTING OF THE SOIL'S PHYSICAL PROPERTIES .....
        WRITE(6,55) NSOIL
        DO 46 I=10,200,10
        U=-1.0*I
        THET = FTH(U,NSOIL)
        COND = FK(U,NSOIL)
        CAP  = FC(U,NSOIL)
        WRITE(6,56) U,THET,COND,CAP
46      CONTINUE
        DO 47 I=300,1400,100
        U=-1.0*I
        THET = FTH(U,NSOIL)
        COND = FK(U,NSOIL)
        CAP  = FC(U,NSOIL)
        WRITE(6,56) U,THET,COND,CAP
47      CONTINUE
        DO 49 I=1500,15500,1000
        U=-1.0*I
        THET = FTH(U,NSOIL)
        COND = FK(U,NSOIL)
        CAP  = FC(U,NSOIL)
        WRITE(6,56) U,THET,COND,CAP
56      FORMAT(2X,F8.1,7X,F5.3,8X,E12.5,7X,E12.5)
49      CONTINUE
```

```
C
C      SOME INTIALIZATIONS
C
       SAVMAS = 0.
       SAVDEN = 0.
       ITIM = 1
       EPS = .001
       IT=50
       TIM = 0.
       DZ = -ZBOT/NZ
       NC = 150
       NZP1 = NZ+1
       IFLAG = 0
       ISWIT = 0
C
C      INITIAL VALUES OF Z,U,V AND TH AT TIME ZERO
50     DO 60 I=1,NZP1
         Z(I) = FLOAT(I-1)*DZ
         UO(I)=UIN(Z(I),NSOIL)
         TH(I)=FTH(UO(I),NSOIL)
60     CONTINUE
       DO 65 I=2,NZP1
            CON = -FK(.5*(UO(I)+UO(I-1)),NSOIL)
            VO(I) = CON*((UO(I)-UO(I-1))/DZ) + CON
65     CONTINUE
       VO(1)=VO(2)
C
C **LIST THE INITIAL VALUES OF DEPTH,THETA,PRESSURE HEAD AND FLUX RESP.
C
       WRITE(6,72)TIM,KOUNT,UTOP,UBOT
       DO 77 I=1,NZP1
       WRITE(6,75) I,Z(I),TH(I),UO(I),VO(I)
77     CONTINUE
C
C
       CTNEW = 0.D+00
       DO 80 I=1,NZ
            CTNEW=CTNEW-(TH(I)+TH(I+1))*DZ
80     CONTINUE
       CTNEWS = CTNEW/2.
       WRITE(6,85) TIM,CTNEWS
       WRITE(6,260)
C
C      STORE VALUES FOR MATRIX SETUP
       N2P4 = 2*(NZ-1)+4
       N=NZ-1
       DO 90 J=1,NZP1
            U1(J)=UO(J)
            V1(J)=VO(J)
90     CONTINUE
```

```
C
C ** MAIN LOOP: NT IS AMOUNT OF TIME STEPS ALLOWED TO REACH 'TEND' *
      DO 800 KOUNT=1,NT
C
C          T R A N S I E N T   T O P -   O R
C
C          B O T T O M   B O U N D A R Y   C O N D I T I O N S
C
           CALL UNSTE(UTOP,UBOT,U1,V1,NZP1)
C
C
100        DO 110 I=1,NZP1
                UO(I)= U1(I)
                TH(I)= FTH(U1(I),NSOIL)
                VO(I)= V1(I)
110        CONTINUE
C
C ***********************************
      IF(IFLOW.EQ.1)GO TO 120
C ***********************************
C
C
C     CHANGE FROM CONSTANT FLUX TO A CONSTANT PRESSURE HEAD BOUNDARY IN-
C     CASE OF EVAP.MODEL AND IF H< -15000 CM  (---> ALPHA1=1 )
      IF(UO(1).GT.-15000.) GOTO 120
      IF(ISWIT.EQ.0) CALL REZEE(Z,U2,V2,U2,V2,NZ,NC)
C     REZEE IS ALWAYS CALLED FOR THE FIRST TIME STEP AFTER CHANGE TO
C     CONSTANT PRESSURE HEAD.
      ISWIT = 1
      ALPH1 = 1.
      ALPH2 = 0.
      UTOP = UO(1)
C
C
120   CONTINUE
C
C     'IT' IS THE MAX. NUMBER OF ITERATIONS ALLOWED IN ONE TIME STEP.
C     THE INPUT MATRIX A IS COMPRESSED IN BAND STORAGE MODE. IT HAS
C     5 COLUMNS AND (2*NZ +2) ROWS.
C
      DO 700 KK=1,IT
      A(1,1) = 0.D+00
      A(1,2) = 0.D+00
      A(1,3) = ALPH1
      A(1,4) = ALPH2
      A(1,5) = 0.D+00
      RH(1)  = UTOP
```

```
      DO 130 I=1,NZ
            DZ= Z(I+1)-Z(I)
            R = DZ/DT
            J = 2*I
            A(J,1) = 0.D+00
            A(J,2) = -FK(.5*(U1(I)+U1(I+1)),NSOIL)
            A(J,3) = +.5*DZ
            A(J,4) = -A(J,2)
            A(J,5) = +.5*DZ
            RH(J)  = A(J,2)*DZ
            J=J+1
            A(J,1) = -.5*R*FC(.5*(U1(I)+U1(I+1)),NSOIL)
            A(J,2) = +1.
            A(J,3) = A(J,1)
            A(J,4) = -1.
            A(J,5) = 0.D+00
            RH(J)  = A(J,1)*(U0(I)+U0(I+1))
130   CONTINUE
      A(N2P4,1) = 0.D+00
      A(N2P4,2) = BETH1
      A(N2P4,3) = BETH2
      A(N2P4,4) = 0.D+00
      A(N2P4,5) = 0.D+00
      RH(N2P4)  = UBOT
      NL=2
      IA=310
      M=1
      IJOB=0
C
C ***********************************************************************
C SUBROUTINE LEQT2B IS MORE ACCURATE THAN LEQT1B
       CALL LEQT1B(A,N2P4,NL,NL,IA,RH,M,IA,IJOB,XL,IER)
C
C      CALL LEQT2B(A,N2P4,NL,NL,IA,RH,M,IA,IJOB,A1,IA,XL,IER)

       IF(IER.EQ.129) WRITE(6,*)TIM
C * ERROR MESSAGE: IER=129 - MATRIX IS SINGULAR
C **               IER=130 - MATRIX A IS TOO ILL CONDITIONED, DOES NOT
C **                         CONVERGE.
C ***********************************************************************
C
      U2(1) = RH(1)
      DO 140 I=1,NZ
           J=2*I
           V2(I)=RH(J)
           U2(I+1) = RH(J+1)
140   CONTINUE
      V2(NZP1)=RH(N2P4)
C
C ** THE FIRST COMPUTATION FOR EACH TIME STEP IS MEMORIZED: US AND VS *
      IF(KK.GT.1)GOTO 160
      DO 150 I=1,NZP1
           US(I) = U2(I)
           VS(I) = V2(I)
150   CONTINUE
C
160   CONTINUE
C ** NO MATTER HOW LARGE THE ERROR IN THE MASS BALANCE IS, TAKE THE LAST
C ** COMPUTED VALUES IF KK EQUALS 'IT'.
      IF(KK.EQ.IT)GOTO 180
```

```
C
C ** CHECK FOR CONVERGENCE. IF THE NEWLY COMPUTED AND THE PREVIOUS
C ** VALUES DIFFER TOO MUCH,TRY AGAIN WITH THE NEW VALUES AS INPUT.
C ** THE CRITERION : UMAX MUST BE SMALLER THAN 0.0005
      UMAX = 0.
      DO 170 I=1,NZP1
            DEN = V2(I)
            IF(DEN.EQ.0.) DEN = 1.
            UMAX = AMAX1(UMAX,ABS((V2(I)-V1(I))/DEN))
            DEN = U2(I)
            IF(DEN.EQ.0.) DEN = 1.
            UMAX=AMAX1(UMAX,ABS((U2(I)-U1(I))/DEN))
170   CONTINUE
      IF (UMAX.GT..0005) GOTO 600
C     CONVERGENGE CRITERION IS MET
      KL = KK
C
180   DO 190 I=1,NZP1
C **        WRITE VALUE IF PRESSURE HEAD IS POSITIVE AND CALCULATE THE
C **        CORRESPONDING CHANGE IN THETA FROM WATER RETENTION DATA.
            IF(U2(I).GE.0.000001) WRITE(6,185)I,U2(I)
            TH(I) = FTH(U2(I),NSOIL)-TH(I)
190   CONTINUE
C
C ** THE FOLLOWING STATEMENTS CHECK THE MASS BALANCE EQUATION.
C ** 'XMB1' IS RELATIVE MASS BALANCE:RELATIVE TO THE DIFFERENCE
C ** IN FLUX BETWEEN THE TOP AND THE BOTTOM OF THE PROFILE.
C
      CTNEW = 0.D+00
      DO 200 I=1,NZ
            CTNEW = CTNEW +(TH(I)+TH(I+1))*(Z(I)-Z(I+1))
200   CONTINUE
      CTNEWS = CTNEW/2.
      SAVE1 = CTNEWS
      WIN=(-V2(1)-V0(1)+V2(NZP1)+V0(NZP1))*DT/2.
      SAVE2 = WIN
      XMB = ABS(CTNEWS-WIN)
      XMB1 =(XMB/ABS(WIN))*100.0
C
C
C THE MAX. PRESSURE HEAD OR FLUX DIFFERENCE OCCURING OVER THE WHOLE
C PROFILE BETWEEN THE FIRST AND THE LAST ITERATION IN THE CURRENT TIME
C STEP IS PRINTED AS AN ERROR CONTROL.
      RMAXK = 0.D+00
      DO 210 I=1,NZP1
            X1 = ABS(U2(I)-US(I))
            X2 = ABS(V2(I)-VS(I))
            RMAXK = AMAX1(X1,X2,RMAXK)
210   CONTINUE
      RMAXK = .5*RMAXK
C
```

```
C ** 'XMB' REFLECTS THE ABSOLUTE MASS BALANCE IN THE CURRENT TIME STEP.
C ** IF 'XMB' IS LESS THAN A PREDEFINED VALUE 'EPS' THE TIMESTEP IS
C ** COMPLETED.
C ** IF HOWEVER 'XMB'> 'EPS' CALL REZEE, WHICH RECOMPUTES THE
C ** GRIDPOINTS DISTANCES. ALSO THE TIME STEP WILL THEN BE HALVED.
C
      WRITE(6,230) TIM,DT,XMB,XMB1,RMAXK,KK
C     REZEE IS ALWAYS CALLED IN THE FIRST TIMESTEP:
      IF(IFLAG.EQ.0) XMB=EPS+1.
      IFLAG =1
      IF(XMB.LE.EPS ) GOTO 250
      CALL REZEE(Z,U1,V1,U0,V0,NZ,NC)
      DO 240 I=1,NZP1
           U1(I)=U0(I)
           V1(I)=V0(I)
240   CONTINUE
      DT = DT/2.
      GOTO 100
C
C
C ***************** READY FOR THE NEXT TIME STEP ******************
250   TIM = TIM + DT
C
C ** SAVMAS AND SAVDEN ARE THE TWO CUMULATIVE COMPONENTS OF THE MASS
C ** BALANCE EQUATION DURING THE COMPUTATIONS.
      SAVMAS = SAVMAS + SAVE1
      SAVDEN = SAVDEN + SAVE2

C ** PRINT VALUES IF 'TIM' EQUALS ANY 'TM(ITIM)'
      IF(TIM.LT.TM(ITIM).OR.ITIM.EQ.(NTOUT+1)) GOTO 500
      ITIM = ITIM+1
C
      CALL REZEE(Z,U2,V2,U2,V2,NZ,NC)
C
C
C ** COMPUTE MOISTURE CONTENTS FROM WATER RETENTION DATA
      DO 300 I=1,NZP1
          TH(I)   = FTH(U2(I),NSOIL)
300   CONTINUE
C

C ** ALL THE COMPUTED VALUES ARE NOW WRITTEN TO DISK
                WRITE(2,280)TIM,DT
                WRITE(2,290)(Z(II),II=1,NZP1)
                WRITE(2,290)(TH(I),I=1,NZP1)
                WRITE(2,290)(U2(I),I=1,NZP1)
                WRITE(2,290)(V2(I),I=1,NZP1)
                WRITE(6,72) TIM,KOUNT,UTOP,UBOT
C
                DO 350 I=1,NZP1
                WRITE(6,75) I,Z(I),TH(I),U2(I),V2(I)
350             CONTINUE
C
                IF(TIM.GT.TEND.OR.ITIM.EQ.(NTOUT+1)) TM(ITIM) = TEND
                WRITE(6,850) SAVMAS,SAVDEN,TM(ITIM)
                IF(TIM.GT.TEND) GO TO 820
                WRITE(6,260)
C
C
C
```

```
C * IF TIM EQUALS TEND THE DESIRED COMPUTATIONS ARE FINISHED *****
C * IF THE MASS BALANCE IS MUCH SMALLER THAN THE PREDEFINED 'EPS'VALUE
C * NEW GRID POINTS DISTANCES ARE CALCULATED IN REZEE AND THE
C * TIME STEP MAY BE LARGER FOR SUBSEQUENT TIME STEPS.
500   IF(TIM.GT.TEND) GO TO 820
      IF(XMB.GT..1*EPS ) GOTO 600
      CALL REZEE(Z,U2,V2,U2,V2,NZ,NC)
      DT= 2.*DT
C
C
600   CONTINUE
C
C * THE COMPUTED VALUES FOR FLUX AND PRESSURE HEAD ARE NOW RESTORED.
      DO 650 I=1,NZP1
           U1(I) = U2(I)
           V1(I) = V2(I)
650   CONTINUE
C
      IF(KK.EQ.IT.OR.UMAX.LT..0005) GO TO 800
C
C
C
700   CONTINUE
800   CONTINUE
C
820   WRITE(6,840) TEND

C
55    FORMAT(1H1' THE PHYSICAL PROPERTIES OF SOILNR ',
     1I3,3(/),' PRESSURE HEAD',3X,
     2'THETA',3X,'HYDRAULIC CONDUCTIVITY',3X,'WATERCAPACITY',
     3/,7X,'CM',27X,'CM/HR',13X,'CM -1',/)
72    FORMAT(1H1'VALUES FOR THETA,PRESSURE HEAD AND FLUX FOR THE',
     1' SUCCESSIVE GRIDPOINTS AT TIME: ',F10.5,' TIMESTEPNR. ',I5,
     2(/),' TOPBOUND.CONDITION    ',F10.5,/,' BOTTOMBOUND.CONDITION ',
     3F10.5,2(/),1X,'GRIDPOINT',
     45X,'DEPTH(CM)',7X,'THETA ',10X,'H(CM)',8X,'FLUX(CM/HR)')
75    FORMAT(2X,I5,4X,4(3X,E12.5))
85    FORMAT(/,' WATER IN PROFILE AT TIME',F7.4,':',F10.5,' CM',3(/))
185        FORMAT(T75,'POS. PRESSURE HEAD AT GRIDPOINT:',I3,E12.5)
230   FORMAT(1X,F10.5,4(E12.5,3X),2X,I4)
260   FORMAT(1H12X,'  TIME',4X,'TIMESTEP',6X,'ABS.MASSBAL.',
     13X,'REL.MASSBAL.',3X,'ERRORCONTROL',4X,'NR. OF ',/,5X,'HRS',
     27X,'HRS',13X,'CM',13X,'PERC',8X,'CM,CM/HR',5X,'ITERATIONS')
280   FORMAT(2F7.4)
290   FORMAT(6F12.5)
840   FORMAT(3(/),' COMPUTATIONS ARE ENDED, TIME IS ',F7.4,' HRS')
850   FORMAT(3(/),' CHANGE IN STORAGE IN THE WHOLE PROFILE',15X,E15.6,
     1' CM',/,1X,'TOTAL FLUX DIFFERENCE BETWEEN TOP AND BOTTOM TILL',
     2' NOW',E15.6,' CM',3(/),' COMPUTATIONS WILL PROCEED TILL ',
     3F7.4,' HOURS')
C
      STOP
      END
C
C
```

```
C
      SUBROUTINE REZEE(Z,DU,DV,U,V,N,NC)
C
C     PURPOSE: TO RECOMPUTE NEW SPACE STEPS
C
      COMMON IFLOW,NSOIL,TIM
C
      DIMENSION Z(NC),U(NC),V(NC),DV(NC),DU(NC)
      DIMENSION UNEW(150),VNEW(150),ZNEW(150),VNEW1(150)
      DIMENSION TH(150),COEFG(2,300),DV1(150),DU1(150)
C
C
      K=N+1
      TH(1)=FTH(DU(1),NSOIL)
      TH(K)=FTH(DU(K),NSOIL)
      DVMAX = 0.
      DO 1 I=1,N
         DZ = Z(I+1)-Z(I)
         DV1(I) = (-DV(I+1)+DV(I))/DZ
         DU1(I) =(+DU(I+1)-DU(I))/DZ
         DK = (FK(DU(I+1),NSOIL)-FK(DU(I),NSOIL))/DZ
         DIVK = FK(.5*(DU(I+1)+DU(I)),NSOIL)
         VNEW(I) = ABS((DV1(I)-DU1(I)*DK-DK)/DIVK)
         VNEW1(I)= (DV1(I)-DU1(I)*DK-DK)/DIVK
1     CONTINUE
C
      IF(TIM.LT..04) GO TO 8
C     Z ---> DEPTH
C     VNEW ---> SECOND DERIVATIVE OF H WITH Z
C     DU1 ---> FIRST DERIVATIVE OF H WITH Z
C     DV1 ---> FIRST DERIVATIVE OF V WITH Z
      WRITE(1,180) TIM
      WRITE(1,200) (Z(I),I=1,N)
      WRITE(1,200) (VNEW1(I),I=1,N)
      WRITE(1,200) (DU1(I),I=1,N)
C     WRITE(1,200) (DV1(I),I=1,N)
C
8     CONTINUE
      DVMIN = .010
      IF(IFLOW.EQ.1) DVMIN=0.
      COEFG(1,1) = 0.
      DO 10 I=1,N
      IF (VNEW(I).LT.DVMIN)VNEW(I)=DVMIN
      COEFG(2,I)=SQRT(VNEW(I))
10    COEFG(1,I+1) = COEFG(1,I)+COEFG(2,I)*(Z(I)-Z(I+1))
      STEP = COEFG(1,N+1)/N
      IF (STEP.LE.0.) GOTO 90
      J = 1
      DO 30 I=2,N
      STEPI = FLOAT(I-1)*STEP
21    IF(J.EQ.N)  GOTO 27
      IF(STEPI.LE.COEFG(1,J+1)) GOTO 27
      J=J+1
      GOTO 21
27    IF(COEFG(2,J).EQ.0) GOTO 29
      ZNEW(I)=Z(J)-(STEPI-COEFG(1,J))/COEFG(2,J)
      GOTO 30
```

```
29      ZNEW(I)=(Z(J)+Z(J+1))/2.
30      CONTINUE
        GOTO 100
90      STEP = (Z(1)-Z(N+1))/FLOAT(N)
        DO 93 I=2,N
93      ZNEW(I) = Z(1)-FLOAT(I-1)*STEP
100     J=1
        DO 110 I=2,N
101     IF(ZNEW(I).GE.Z(J+1)) GOTO 127
        J=J+1
        GOTO 101
C
C       LINEAIR INTERPOLATION:
C       H AND V ARE DETERMINED FROM THE NEW COMPUTED GRID POINTS.
127     R=(ZNEW(I)-Z(J))/(Z(J+1)-Z(J))
        UNEW(I)=U(J)+(U(J+1)-U(J))*R
        VNEW(I)=V(J)+(V(J+1)-V(J))*R
110     CONTINUE
        DO 120 I=2,N
        U(I)=UNEW(I)
        IF(U(I).GE.0.)WRITE(6,123)I,U(I)
123     FORMAT(1X,' IN REZEE',I5,E20.4)
        V(I)=VNEW(I)
120     Z(I)=+ZNEW(I)
C
        IF(TIM.LT..04) GO TO 250
        DO 132 I=2,N
        TH(I)=FTH(U(I),NSOIL)
132     CONTINUE
        WRITE(1,200) (Z(I),I=1,K)
        WRITE(1,200) (TH(I),I=1,K)
180     FORMAT(F7.4)
200     FORMAT(6E12.5)
C
250     RETURN
        END
C
C
```

```
C
      FUNCTION FTH(H,NSOIL)
C ** COMPUTES WATER CONTENT FROM WATER RETENTION CURVES.
      REAL M,TE,N
      IF(NSOIL.EQ.3) GO TO 10
      IF(NSOIL.EQ.1) GO TO 5
C   SOILNR. 2
      IF(H.GT.-.01) GOTO 2
      IF(H.LT.-7320.) GOTO 1
      BE = (131./ABS(H))**.468
      GAM =(.433-.0760)/(.433+.0760)
      FTH = .433*(COSH(BE)-GAM)/(COSH(BE)+GAM)
      RETURN
1     FTH = (1.71E-02/ABS(H))**.1964
      RETURN
2     FTH = .433
      RETURN
C
C   SOILNR.1
5     FTH=1.611E+06*.212/(1.611E+06+ABS(H)**3.96)+.075
      RETURN
C
C   SOILNR. 3
10    IF(H.GT.-.0001) GO TO 15
      N=3.57168
      ALP=0.02912
      M=1-(1.0/N)
      TE=(1.0/(1.0+(ALP*ABS(H))**N))**M
      FTH=0.296*TE+0.069
      RETURN
15    FTH=0.365
      RETURN
      END
C
C
```

```
C
      FUNCTION FK(H,NSOIL)
C ** HYDRAULIC CONDUCTIVITY VALUES FROM PRESSURE HEAD DATA.
      REAL N,M,TE,ME,KK
      IF(NSOIL.EQ.3) GO TO 20
      IF(NSOIL.EQ.1) GO TO 15
C   SOILNR. 2
      IF(H.GT.-.01) GOTO 2
      IF(H.LT.-7320.) GOTO 1
      BE = (131./ABS(H))**.468
      GAM =(.433-.0760)/(.433+.0760)
      T = .433*(COSH(BE)-GAM)/(COSH(BE)+GAM)
      GOTO  10
1     T = (1.71E-02/ABS(H))**.1964
      GOTO  10
2     T = .433
10    CONTINUE
      FK = 5.4E-08*EXP(54.29*T)/24.
      RETURN
C
C  SOILNR. 1
15    FK=34.*1.175E+06/(1.175E+06+ABS(H)**4.74)
      RETURN
C
C  SOILNR. 3
20    IF(H.GT.-.0001) GO TO 30
      N=3.57168
      ALP=0.02912
      M=1-(1.0/N)
      TE=(1.0/(1+(ALP*ABS(H))**N))**M
      ME=1.0/M
      KK=(1-(TE**ME))**M
      FK=10.95*SQRT(TE)*((1-KK)**2)
      RETURN
30    TE=1
      FK=10.95
      RETURN
      END
C
C
```

```
C
      FUNCTION FC(H,NSOIL)
C ** WATER CAPACITY VALUES FROM PRESSURE HEAD DATA.
      REAL N,M,R,T,HH
      IF(NSOIL.EQ.3) GO TO 10
      IF(NSOIL.EQ.1) GO TO 5
C
C   SOILNR. 2
      IF(H.GT.-.01) GOTO 2
      IF(H.LT.-7320.) GOTO 1
      BE = (131./ABS(H))**.468
      GAM = (.433-.0760)/(.433+.0760)
      DBE = .468/131.*(131./ABS(H))**1.468
      FC = .433*SINH(BE)*2.*GAM*DBE/(COSH(BE)+GAM)**2
      RETURN
1     FC = .1964/1.71E-02*(1.71E-02/ABS(H))**1.1964
      RETURN
2     FC = 0.
      RETURN
C
C  SOILNR. 1
5     FC=1.611E+06*.212*3.96*ABS(H)**2.96
      FC = FC/(1.611E+06+ABS(H)**3.96)**2
      RETURN
C
C  SOILNR. 3
10    IF(H.GT.-.0001) GO TO 15
      ALP=0.02912
      N=3.57168
      M=1-(1.0/N)
      R=-M-1
      T=N-1
      HH=(1+(ALP*ABS(H))**N)**R
      FC=0.296*M*HH*N*(ALP**N)*(ABS(H)**T)
      RETURN
15    FC=0
      RETURN
      END
C
C
```

```
C
      FUNCTION UIN(Z,NSOIL)
C ** THE INITIAL CONDITIONS, EXPRESSED IN PRESSURE HEAD VALUES AS A
C ** FUNCTION OF DEPTH.
      IF(NSOIL.EQ.1) GO TO 5
      IF(NSOIL.EQ.3) GO TO 10
      UIN = -1.
      RETURN
5     UIN = -61.5
      RETURN
10    UIN = -26.000
      RETURN
      END
C
C
C
      SUBROUTINE UNSTF(UTOP,UBOT,U1,V1,NZP1)
C
C   PURPOSE: TO ACCOUNT FOR TRANSIENT TOP OR BOTTOM BOUNDARY CONDITIONS.
C
      COMMON IFLOW,NSOIL,TIM
      DIMENSION U1(150),V1(150)
      IF (TIM.EQ.0) GO TO 20
C     UBOTA=U1(NZP1)
C     UBOT = -FK(UBOTA,NSOIL)
      UBOT= (1+0.0*TIM)*UBOT
      UTOP= UTOP
20    RETURN
      END
```

****A   END   JOB 8751   AYL59JH5   6201   17 PAGES   BIN   124   10.05.01 AM 0:

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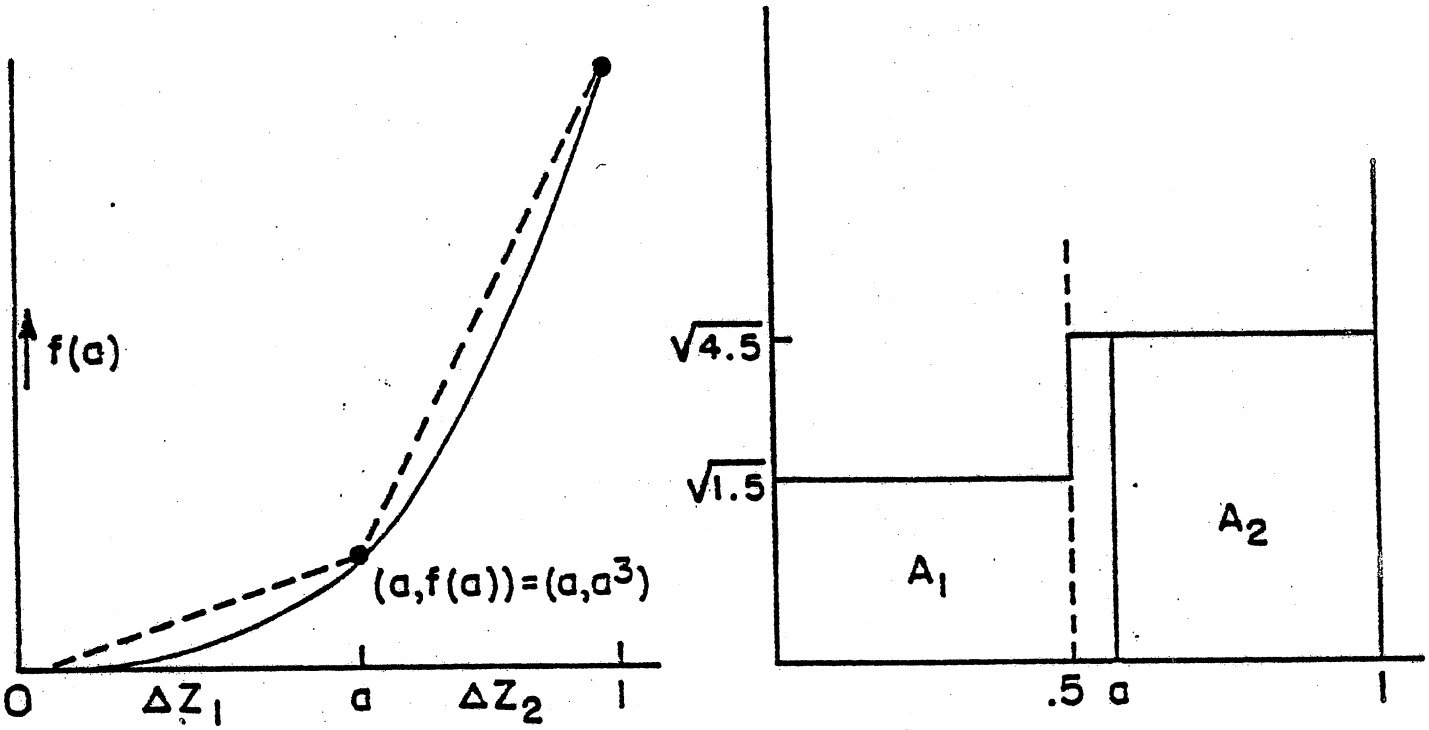
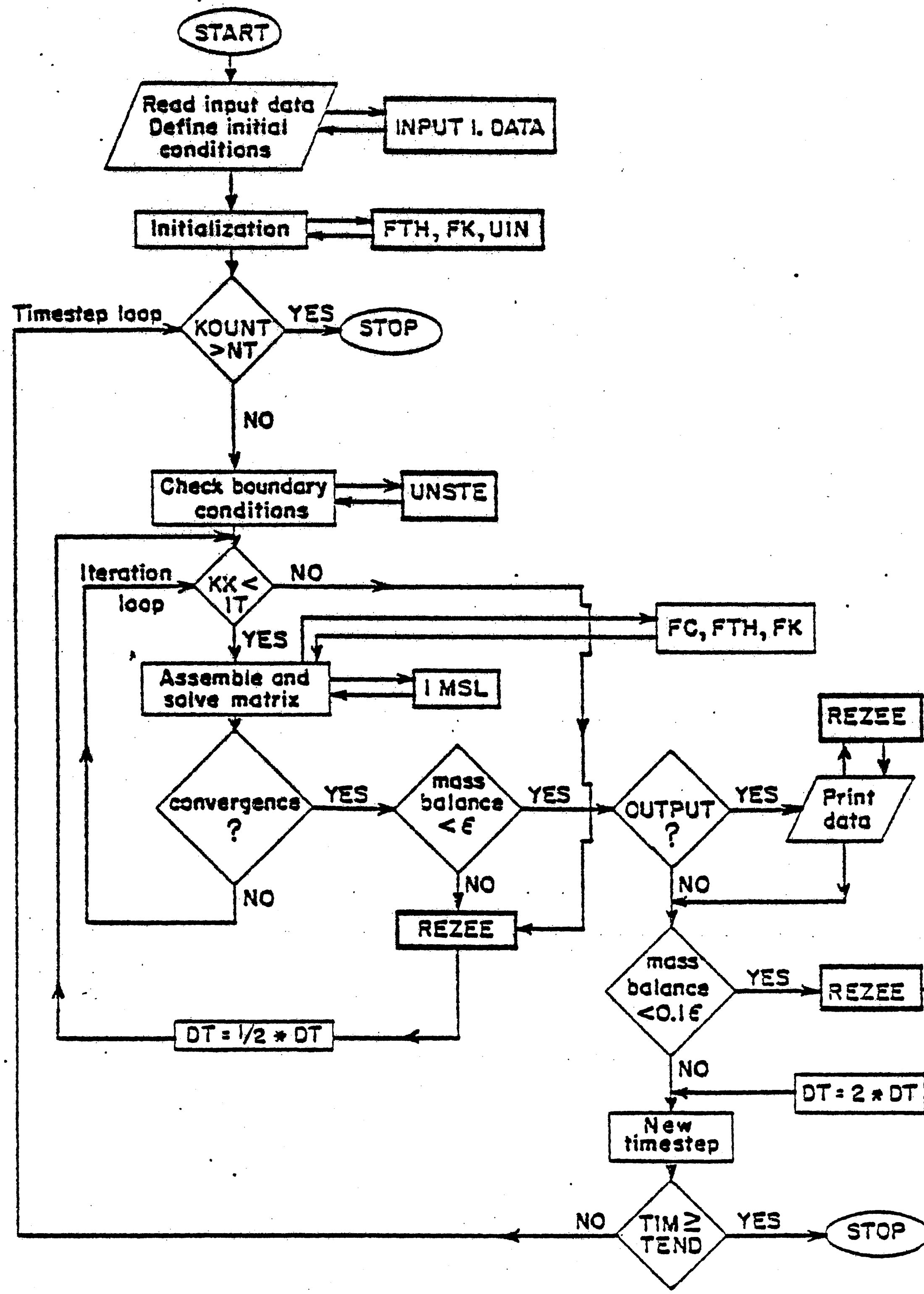


APPENDIX FIG. 1. Illustrations for selection of z_i.



APPENDIX FIG. 2. Flowchart of WAFLOW.